PLANE FLOWS WITH FEW STAGNATION POINTS[1]

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Introduction. This announcement deals with the subject of continuous flows in the Euclidean plane. It has long been known (cf. [1]) that every closed set in the plane is the invariant set of some continuous flow. In a recent paper [2], this author has shown that the same statement is not true for flows all of whose orbits are closed in the plane. In fact a necessary and sufficient condition is given for a closed set to be the invariant set of such a flow.

The author here announces new results in the same direction. The flows now dealt with are more general than those with closed orbits. They include flows with no stagnation points (see definition below), finitely many stagnation points, and countably many stagnation points. Results similar to those for flows with closed orbits are obtained for flows with no stagnation points or finitely many. On the other hand, every closed set can be exhibited as the invariant set of a flow with countably many stagnation points.

DEFINITIONS. A continuous flow in a topological space X is a continuous mapping ϕ from $R\times X$ onto X which satisfies the group property:

$$\forall t_1, t_2 \in R, \qquad \forall x \in X, \qquad \phi(t_1, \phi(t_2, x)) = \phi(t_1 + t_2, x).$$

For each $t\in R$, the mapping $\phi_{(t)}$ defined by $\phi_{(t)}(x) = \phi(t, x)$ is a homeomorphism, and these homeomorphisms form a topological group which is a continuous homomorphic image of the real line.

For each $x\in X$, we define the set $\mathcal{O}(x) = \{\phi(t, x) \mid t\in R\}$, called the *orbit* of x. If $\mathcal{O}(x) = \{x\}$, then x is called an *invariant point* of ϕ, and we denote the set of invariant points of ϕ as $F(\phi)$, the *invariant set* of ϕ. For each $x\in X$, $\mathcal{O}(x)$ is either a single point, a simple closed curve, or a 1-1 continuous image of the real line. We call these last two a *circle* and a *line* respectively, using the terms *genuine circle* and *straight line* when we mean these. Suppose $\mathcal{O}(x)$ is a line, and we can find a point $y\in X$ and a sequence $\{t_n\}$ converging to $+\infty$ (resp. $-\infty$) such that $\phi(t_n, x)\rightarrow y$. Then y is called an *endpoint of* $\mathcal{O}(x)$ *at the positive* (resp. *negative*) *end*. If $y = \lim_{t\rightarrow+\infty} \phi(t, x)$, then y is a *strong endpoint* (*at the positive end*), with a similar definition at the negative end.

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