A NEW LOCAL PROPERTY OF EMBEDDINGS

BY O. G. HARROLD[1]

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It is known that the possible embeddings of a topological $n-1$ manifold M^{n-1} in the euclidean space E^n differ in the cases $n=3$ and $n>3$ in a curious way. A topological $n-1$ sphere can fail to be locally flat at an arbitrary finite number of points if $n=3$. For $n>3$ this cannot happen at a set consisting of a single point [2]. It is unresolved if an S^{n-1} in E^n can fail to be locally flat at a pair of points. In this note we introduce a new notion, described in detail below, called a *locally weakly flat* embedding and show that if a manifold M^{n-1} in E^n is locally flat at each point except possibly at the points of a finite set Y and if M^{n-1} is locally weakly flat at each point of Y, then M^{n-1} is in fact locally flat at every point. In the concluding paragraph an unsolved problem is posed.

Let $p \in M^k \subset E^n$, or more generally $M^k \subset M^n$. Suppose $\epsilon > 0$. Let B^n_ϵ be a ball of diameter less than ϵ whose interior contains p. For $0 < t \leqq \epsilon$ let B_t denote a ball whose interior contains p and is concentric to B^n_ϵ, i.e., regard B_t as a topological product $S^{n-1} \times [0, t]$ with $S^{n-1} \times [0]$ identified with p. For all t such that $\epsilon - t$ is sufficiently small we hypothesize that $\dot{B}_t \cap M$ is a $k-1$ sphere such that the pairs

$$(E^n, \dot{B}^n_t \cap M^k \times I^{n-k+1}) \approx (E^n, S^{k-1} \times I^{n-k+1})$$

are homeomorphic. If for a sequence of positive numbers $\epsilon_1, \epsilon_2, \cdots$ converging to zero, this condition holds, we describe the embedding by saying M^k is locally weakly flat at p. If this holds for all $p \in M^k$, M^k is locally weakly flat in M^n, denoted by LWF.

A comparison with other local properties of embeddings [3] shows that LF = LU ⇒ LWF ⇒ LSPU ⇒ LPU.

For $n=3$, $k=2$ these implications may be reversed [4]. There are examples, for $n=3$, that show that at a single point, local peripheral unknottedness, or local weakly flatness does not imply local flatness [5].

For $n=3$, $k=1$, LU and LPU are entirely independent. In this paper attention is restricted to $k=n-1$.

THEOREM. *Let $M^{n-1} \subset E^n$ be a closed $n-1$ manifold that is locally*

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