

RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited. Manuscripts more than eight typewritten double spaced pages long will not be considered as acceptable.

COMPLETELY 0-SIMPLE AND HOMOGENEOUS n REGULAR SEMIGROUPS

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1. In this note we state three new results (Theorems 1, 4 and 5) about the completely 0-simple and homogeneous n regular semigroups.

We follow the notation and terminology of [1] unless stated otherwise. Throughout, S denotes a semigroup with zero. Let $a \in S \setminus 0$. Denote by $V(a)$ the set of all inverses of a in S , that is, $V(a) = \{x \in S: axa = a, xax = x\}$. A semigroup S with zero is said to be homogeneous n regular if the cardinal number of the set $V(a)$ of all inverses of a is n for every nonzero element a in S , where n is a fixed positive integer. Let T be a subset of S . We denote by $E(T)$ the set of all idempotents of S in T .

2. The next theorem is a generalization of R. McFadden and Hans Schneider's theorem [3].

THEOREM 1. *Let S be a 0-simple semigroup and let n be a fixed positive integer. Then the following are equivalent.*

(i) S is a homogeneous n regular and completely 0-simple semigroup.
(ii) For every $a \neq 0$ in S there exist precisely n distinct nonzero elements $(x_i)_{i=1}^n$ such that $ax_i a = a$ for $i = 1, 2, \dots, n$ and for all c, d in S $cdc = c \neq 0$ implies $dcd = d$.

(iii) For every $a \neq 0$ in S there exist precisely n distinct nonzero idempotents $(e_i)_{i=1}^h = E_a$ and k distinct nonzero idempotents $(f_j)_{j=1}^k = F_a$ such that $e_i a = a = a f_j$ for $i = 1, 2, \dots, h, j = 1, 2, \dots, k, hk = n$, E_a contains every nonzero idempotent which is a left unit of a , F_a contains every nonzero idempotent which is a right unit of a and $E_a \cap F_a$ contains at most one element.

(iv) For every $a \neq 0$ in S there exist precisely k nonzero principal right ideals $(R_i)_{i=1}^k$ and h nonzero principal left ideals $(L_j)_{j=1}^h$ such that