RADON-FOURIER TRANSFORMS ON SYMMETRIC SPACES AND RELATED GROUP REPRESENTATIONS[1]

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In §2 we announce some results in continuation of [10], connected with the Radon transform. §1 deals with tools which also apply to more general questions and §§2–3 contain some applications to group representations. A more detailed exposition of §2 appears in Proceedings of the U. S.-Japan Seminar in Differential Geometry, Kyoto, June, 1965.

1. **Radial components of differential operators.** Let V be a manifold, v a point in V and V_v the tangent space to V at v. Let G be a Lie transformation group of V. A C^∞ function f on an open subset of V is called locally invariant if $Xf=0$ for each vector field X on V induced by the action of G.

Suppose now W is a submanifold of V satisfying the following transversality condition:

(T) For each $w \in W$, $V_w = W_w + (G\cdot w)_w$ (direct sum).

If f is a function on a subset of V its restriction to W will be denoted $\bar{f}$.

LEMMA 1.1. *Let D be a differential operator on V. Then there exists a unique differential operator $\Delta(D)$ on W such that*

$$(Df)^- = \Delta(D)\bar{f}$$

for each locally invariant f.

The operator $\Delta(D)$ is called the *radial component* of D. Many special cases have been considered (see e.g. [1, §2], [4, §5], [5, §3], [7, §7], [8, Chapter IV, §§3–5]).

Suppose now dv (resp. dw) is a positive measure on V (resp. W) which on any coordinate neighborhood is a nonzero multiple of the Lebesgue measure. Assume dg is a bi-invariant Haar measure on G. Given $u \in C_c^\infty(G \times W)$ there exists [7, Theorem 1] a unique $f_u \in C_c^\infty(G\cdot W)$ such that

$$\int_{G\times W} F(g\cdot w)u(g, w)\, dg dw = \int_V F(v)f_u(v)\, dv \qquad (F \in C_c^\infty(G\cdot W)).$$

Let $\phi_u \in C_c^\infty(W)$ denote the function $w \to \int u(g, w)dg$.

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