ON THE COUSIN PROBLEMS[1]

BY AVNER FRIEDMAN

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It is well known that if Ω is a domain of holomorphy in C^n then it is a Cousin I domain; it is also a Cousin II domain if and only if $H^2(\Omega, Z)=0$. In this work we prove that some general classes of domains which are not domains of holomorphy are both Cousin I and Cousin II domains. Recall that Ω is Cousin I (II) if and only if $H^1(\Omega, \mathcal{O})=0$ ($H^1(\Omega, \mathcal{O}^*)=0$) where $\mathcal{O}$ is the sheaf of germs of holomorphic functions under addition and $\mathcal{O}^*$ is the sheaf of germs of nowhere-zero holomorphic functions under multiplication. If $H^1(\Omega, Z)=0$ then "Ω Cousin II" implies "Ω Cousin I" and if $H^2(\Omega, Z)=0$ then "Ω Cousin I" implies "Ω Cousin II."

In what follows we take $n\geqq 3$ since, for $n=2$, Ω is Cousin I if and only if Ω is a domain of holomorphy [1].

DEFINITIONS. An open relatively compact set A in a complex manifold X is called *q-convex* if $A=\{z;\ z\in A_0,\ \phi(z)<0\}$ where A_0 is a neighborhood of $\bar{A}$, ϕ is twice continuously differentiable in A_0, grad $\phi\neq 0$ on ∂A, and the Levi form on ∂A has at least $n-q+1$ positive eigenvalues. If A and B are q-convex, $B\subset A$, and if there exists a function $\phi(z, t)$ ($z\in A_0$, $0\leqq t\leqq 1$) twice continuously differentiable in z such that the sets $D_t=\{z;\ z\in A_0,\ \phi(z, t)<0\}$ are q-convex and lie in A_0 and $D_0=A$, $D_1=B$, then we say that A and B are *q-convex homotopic*. Example: if A, B are strictly convex then they are 1-convex homotopic.

Let K_1, L_1 be open convex sets in the z_1-plane, $0\in L_1$, $\bar{L}_1\subset K_1$, and set $A_1=K_1\backslash\bar{L}_1$. Let $K'=K_2\times\cdots\times K_n$, $L'=L_2\times\cdots\times L_n$ be open convex generalized polydiscs (K_j, L_j lie in the z_j-plane) with $0\in L'$, $\bar{L}'\subset K'$. All the previous sets are taken to be bounded. Set $G_0=A_1\times K'$, $G_1=K_1\times(K'\backslash\bar{L}')$, $G=G_0\cup G_1$.

LEMMA 1. *G is both Cousin* I *and Cousin* II.

The proof that G is Cousin I is a straightforward generalization of the proof of [7, Hilfsatz]. Thus, it remains to show that $H^2(G, Z)=0$.

LEMMA 2. *$H^r(G, Z)=0$ for $0<r\leqq 2n$.*

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