

# INCONSISTENT HOMOGENEOUS LINEAR INEQUALITIES

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If the set of linear inequalities

$$\sum_{j=1}^N a_{ij}x_j > 0, \quad i = 1, 2, \dots, M,$$

is consistent, there is an open convex polyhedral region of  $E^N$ , any point of which represents a solution vector  $x$  for the system. Iterative methods for finding a solution point have been given by Agmon [1] and Novikoff [2], among others.

If the set is inconsistent no such region exists. A generalization of the concept of solution is a collection of vectors  $x^{(1)}, x^{(2)}, \dots, x^{(2k+1)}$ , or "committee," such that each inequality is satisfied by a majority of the members of the committee. This notion has application in pattern recognition [3].

The set of inequalities is contradictory if two of the inequalities represent half spaces separated by the same plane. A simple geometric argument shows that a committee solution exists for any noncontradictory set of homogeneous linear inequalities. The proof will be published elsewhere [4].

## REFERENCES

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3. N. J. Nilsson, *Learning machines*, McGraw-Hill, New York, 1965.
4. C. M. Ablow and D. J. Kaylor, *A committee solution of the pattern recognition problem*, *IEEE Trans. Information Theory*, 1965.

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