KERNEL FUNCTIONS AND NUCLEAR SPACES

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As is well known, it is possible to represent any complete, locally convex space E as

$$(1) \qquad E = \underset{\leftarrow \alpha \in A}{\text{proj}} E_\alpha,$$

where the E_α are Banach spaces. If the projective mappings of (1) are nuclear [2], E is called a nuclear space. For complete spaces, this definition is equivalent to Grothendieck's original one (see [2], [8]). It is possible to treat nuclearity for countable inductive limit spaces in a dual fashion:

DEFINITION 1. E is called an (LN)-space if

(1) $E = \text{ind}_{\rightarrow n} E_n$, $n = 1, 2, \cdots$, where the E_n are Banach spaces,

(2) the inductive mappings (imbeddings) are nuclear.

We have the following theorem.

THEOREM 1. *Every* (LN)*-space is nuclear* (*in the sense of Grothendieck*).

For regular inductive limit spaces the inverse theorem is also true (a space $E = \text{ind}_{\rightarrow n} E_n$ is called regular, if every bounded set $A \subset E$ is already bounded in some E_{n_0}).

THEOREM 2. *If the regular space $E = \text{ind}_{\rightarrow n} E_n$, the E_n being Banach spaces, is nuclear, then it is an* (LN)*-space.*

In the above definitions and theorems, we can without loss of generality substitute Hilbert spaces H_n for the Banach spaces E_n and Hilbert-Schmidt mappings for nuclear mappings.

In what follows we need the concept of a reproducing kernel. We quote the definition of Aronszajn [1], [6]. Let H be a Hilbert space with scalar product $(\, , \,)_x$, consisting of functions $f(x)$ defined on some point set G. The function $K(x, y)$, $x \in G$, $y \in G$ is called a reproducing kernel if:

(1) for every fixed y the function $K(x, y)$ of x belongs to H,

(2) $K(x, y)$ has the reproducing property

$$f(y) = (f(x), K(x, y))_x \quad \text{for all } f \in H.$$

THEOREM 3. *Let H_n, $n = 1, 2, \cdots$, be a sequence of Hilbert spaces with reproducing kernels $K_n(x, y)$, where the scalar product in H_n is*