A FIELD OF COHOMOLOGICAL DIMENSION 1 WHICH IS NOT C_1

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Let k be a field of characteristic q (= prime or 0) and let r be a nonnegative integer. Then k is said to be C_r if and only if every (homogeneous) form of degree d in n variables over k has a nontrivial zero over k if $n>d^r$. In Serre [4, Chapitre II, Corollaire to Proposition 8] the following result is obtained: If k is C_1 then $\dim(k)\leqq 1$ and $[k: k^q]=1$ or q. Here $\dim(k)$ is defined cohomologically. Serre then remarks: "On ignore si la réciproque du corollaire précédente est vrai-c'est peu probable."

Actually, the problem of the relation of cohomological dimension r and C_r had been previously raised in Serre [3]. We exhibit below a field R of characteristic zero of dimension 1 which is not C_1. This implies, for all $r\geqq 1$, the existence of fields of dimension r which are not C_r. But the situation is worse than that: R is quasi-finite in the sense of Serre [2, Chapitre XIII, §2], and for all r, R is not C_r. The interest in these considerations stemmed from a possible relation with Artin's conjecture which states: If k is a totally imaginary number field or a p-adic field, then k is C_2. Indeed, for such fields k, $\dim(k)=2$ as is proved in Serre [4, Chapitre II, Corollaire to Proposition 12, Proposition 13].

We now define R. Let F be an algebraically closed field of characteristic zero. If K is a field, then $K((t))$ denotes the field of formal power series in t over K. Let $F_2=F((t_2))(t_2^{1/n}: 2\nmid n)$. If p is a prime greater than 2 and q is the largest prime less than p, we recursively define $F_p=F_q((t_p))(t_p^{1/n}: p\nmid n)$. Finally we set $R=\text{inj}\lim_p F_p$.

THEOREM. *R is quasi-finite, but R is not C_r for any r.*

The proof will appear in Ax [1].

REFERENCES

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4. ———, *Cohomologie Galoisienne*, Mimeographed notes, College de France, 1963 (also as Lecture Notes in Mathematics, No. 5, Springer, Berlin, 1964).

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