RESEARCH PROBLEMS

12. Richard Bellman: *Matrix theory*

All matrices that appear are $N \times N$ symmetric matrices, $N \geqq 2$, and $A \geqq B$ signifies that $A - B$ is nonnegative definite. Consider the set of matrices X with the property that $X \geqq A_1$ and $X \geqq A_2$ where A_1 and A_2 are two given matrices. Let us choose an element in this set (or the element) with the property that $g(X)$, a prescribed scalar function of X, is minimized. For example, $g(X)$ might be $\mathrm{tr}(X)$, $\mathrm{tr}(X^2)$, or the largest characteristic root of X.

This procedure defines a function of A_1 and A_2 which we denote by $m(A_1, A_2)$. Similarly, we may define $m(A_1, A_2, A_3)$, and, generally, $m(A_1, A_2, \cdots, A_k)$ for any $k \geqq 2$.

Can we find a function $g(X)$ with the property that $m(A_1, A_2, A_3) = m(A_1, m(A_2, A_3))$? If so, determine all such functions, and for general $k \geqq 3$ as well. (Received April 10, 1965.)

13. Richard Bellman: *Differential approximation*

Let $y(t)$ be a given vector function belonging to $L^2(0, T)$ and let $x(t)$ be determined as the solution of the linear vector differential equation $x' = Ax$, $x(0) = c$. Under what conditions on c and y does the expression $\int_0^T (x-y, x-y)\,dt$ possess a minimum rather than an infimum with respect to the constant matrix A? (Received May 12, 1965.)

14. Richard Bellman: *A limit theorem*

It is well known that if $u_n \geqq 0$ and $u_{m+n} \leqq u_m + u_n$, for $m, n = 0, 1, \cdots$, then $u_n \sim nc$ as $n \to \infty$ for some constant c. Let $u_n(p)$ be a function of p for $p \in S$, a given set, and $T(p)$ be a transformation with the property that $T(p) \in S$ whenever $p \in S$, $i = 1,2$. Suppose that

$$u_{m+n}(p) \leqq u_m T_1(p) + u_n(T_2(p))$$

for all $p \in S$ and $m, n = 0, 1, \cdots$. Under what conditions on $T_1(p)$ and $T_2(p)$ and S is it true that $u_n(p) \sim ng(p)$ as $n \to \infty$? When is $g(p)$ independent of p? (Received May 12, 1965.)

15. Richard Bellman: *Generalized existence and uniqueness theorems*

Given a second-order linear differential equation $u'' + p(t)u' + q(t)u = 0$, subject to various initial and boundary conditions, there are two types of problems we can consider. The first are the classical