SOME SPACES WHOSE PRODUCT WITH E^1 IS E^4

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1. **Introduction.** If A is a collection of subsets of E^3, then $A^* = \bigcup\{a \mid a \in A\}$. A sequence A_i, $i = 1, 2, 3, \cdots$, of locally finite disjoint collections of subsets of E^3 is trivial if $A_{i+1}^* \subset \operatorname{Int}(A_i^*)$, each element of A_i is a cube with handles semi-linearly imbedded in E^3, and the inclusion map $j: a' \to a$, where $a' \subset a \in A_i$ and $a' \in A_{i+1}$, is null homotopic.

If A_i, $i = 1, 2, \cdots$, is a trivial sequence let G be the set of points of $E^3 - \bigcap A_i^*$ and components of $\bigcap A_i^*$. Let X be the corresponding decomposition space. The main result, Theorem 2, may now be stated.

THEOREM 2. *If each element of* A_i, $i = 1, 2, \cdots$, *is a solid torus, then* $X \times E = E^4$.

This theorem is parallel to results in [1], [3], [4] and others. The proof is similar to that given in [4].

2. **Some useful maps.** Let $D = \{z \mid z \in E^2 \text{ and } |z| \leq 1\}$, $S = \{z \mid z \in E^2 \text{ and } |z| = 1\}$, $D_1 = \{z \mid z \in E^2 \text{ and } |z| \leq 1/2\}$, $T = D \times S$ and $B = D_1 \times S \subset T$. Let $p: D_1 \times E \to B$ be the universal covering of B where p is given by $p(x, t) = (x, e^{it})$ for $x \in D_1$, $t \in E$. Let $h: D_1 \times E \to T \times E$ by $h(x, t) = (x, e^{it}, t)$ and $q: T \times E \to T$ by $q(x, s, t) = (x, s)$ where $x \in D$, $s \in S$ and $t \in E$. Hence $qh(x, t) = p(x, t)$.

Let B' be a finite subcomplex of Int (B) such that the inclusion map $j: B' \to \operatorname{Int}(B)$ is null homotopic. Using the homotopy lifting theorem, there exists $j^*: B' \to D_1 \times E$ such that:

$$\begin{array}{cccc} & D_1 \times E & \xrightarrow{h} & T \times E \\ j^* \nearrow & \downarrow p & & \downarrow q \\ B' \xrightarrow[j]{} & B & \subset & T \end{array}$$

is commutative and both j^* and h are homeomorphisms.

If $u \in B'$, $hj^*(u) = (u, \psi(u))$ where $\psi: B' \to E$. If $(x, s) \in B'$ where $x \in D_1$, $s \in S$, then $j^*(x, s) = (x, w(x, s))$ where $w: B' \to E$. By commutativity,

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