SOME HOMOTOPY GROUPS OF STIEFEL MANIFOLDS[1]

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Paechter [7] made some computations of $\pi_{k+p}(V_{k+m,m})$ where $V_{k+m,m}$ is the Stiefel manifold of m frames in $k+m$ space. In this note we give a table (Table 1) extending his results in the case where m is large. Since $V_{k+m,m}\to V_{k+m+1,m+1}\to S^{k+m}$ is a fibering it is clear that $\pi_{k+p}(V_{k+m,m})$ depends only on k and p for $p\leqq m-2$. This is called the stable range and we feel that these stable groups are the most important ones. On the other hand for small values of m, one of us [4] has made extensive computations and the results are available.

James' periodicity [5, Theorem 3.1] is reflected in the table but the basic periodicity of period 8 is also present.

In [1] it is proved that if $n > 12$, then $\pi_j(SO(n)) = \pi_j(SO) + \pi_{j+1}(V_{2n,n})$ for $j<2n-1$. Hence it is easy to deduce the first fourteen nonstable groups of $SO(n)$ from this table.

Tables of homotopy groups are much more useful if generators are given. Instead of generators we settle for giving the order of the image of $i_*: \pi_{k+p}(S^k)\to\pi_{k+p}(V_{k+m,m})$ (Table 2). One can construct the generators from this information and this map has important connections with Whitehead products [2].

The groups have been computed by using modified Postnikov towers [6]. An outline of the computation for one case, 6 mod 32, is given. The case $k\equiv 6$ mod 32. This procedure is essentially the same as the Adams spectral sequence method.

Let $k=32n+6$ and we suppose m is large. Consider the fibering $V_{32n+6,7}\to V_{32n+m,m+1}\to V_{32n+m,m-6}$. We are only interested in groups in the homotopy stable range so that we can construct a new fibering

$$\Sigma^{-1}V_{32n+m,m-6}\to V_{32n+6,7}\to V_{32n+m,m+1}.$$

We will build the modified Postnikov tower to this fibering. By [3] the cohomology of $V_{32n+m,m+1}$ is given by

$$\begin{aligned} H^i(V_{32n+m,m+1}; Z_2) &= 0, && 0 < i < 32n-1,\\ &= Z_2, && 32n-1\leqq i\leqq 32n+m-1.\end{aligned}$$

Let h_i generate $H^i(V_{32n+m,m+1}; Z_2)$ when it is nonzero. Then $Sq^j h_i$

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