THE EXTREMAL FUNCTIONS FOR CERTAIN PROBLEMS CONCERNING SCHLICHT FUNCTIONS

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1. Let S denote the classical family of schlicht functions on the unit disk normalized by the conditions $f(0)=0$, $f'(0)=1$. Under a suitable metric such as $d(f, g)=\sup\{f(z)-g(z)| : |z|=1/2\}$ it is a compact metric space. Let $0<r<1$. We are interested in the closed subspaces $B_r=\{f\in S: |f(z)|<1/r\}$ and $C_r=\{f\in S: f(z)\notin D(f)\}$, where $D(f)$ is a domain of outer conformal radius $1/r$ with respect to the point at infinity. The general problem is to determine the explicit region of values $V(T)$ of certain continuous functions T from one of these spaces F into some manifold M. We also ask what (extremal) functions in F are mapped by T into $\partial V(T)$, the boundary of $V(T)$. In particular consider the function

$$(*)\quad T(f) = (f^0(z_1), f^1(z_1), \cdots, f^{n_1}(z_1), \cdots, f^0(z_m), \cdots, f^{n_m}(z_m)),$$

where $f^k(z_j)=H(f, z_j, k)$ denotes the value of the kth derivative of f at z_j, except that, for technical reasons H is interpreted as a continuous function into the logarithmic covering surface when $z_j\neq 0$ and $k=0$ or 1.

The well-known results for the case $F=S$, $m=1$, $z_1=0$, due to Spencer and Schaeffer, can be found in [10]. Royden [11] indicated the more general result when $F=S$. Their key tools were Teichmüller's Theorem [10, p. 93] and their variational method. By using Jenkins' General Coefficient Theorem [7] and a form of the Brouwer Fixed Point Theorem we are able to generalize some of their results to a somewhat wider class of functions T and spaces F.

2. For the functions T defined by (*) there are certain quadratic differentials $P(w)dw^2$, indicated by the Teichmüller Principle [8, p. 48], which we call *admissible with respect to* T. We call the pair $(P(w)dw^2, f(z))$ an *admissible association with respect to* T if $P(w)dw^2$ is admissible with respect to T, $f\in F$, $f(\{|z|<1\})$ is an admissible domain with respect to $P(w)dw^2$ in the sense of Jenkins [8, p. 49], and $\{f(z_j): 1\leqq j\leqq m\}$ contains the poles of $P(w)dw^2$ in $f(\{|z|<1\})$.

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