K-SAMPLE ANALOGUES OF RÉNYI'S KOLMOGOROV-SMIRNOV TYPE THEOREMS

BY MIKLÓS CSÖRGÖ[1]

Communicated by J. L. Doob, March 19, 1965

1. Let ξ_{ji}, $i=1, \cdots, n_j$, $j=1, \cdots, k$, be k mutually independent samples of mutually independent random variables having a common continuous distribution function $F(x)$. Let $F_{n_j}(x)$, $j=1, \cdots, k$, be the corresponding empirical distribution functions, that is $F_{n_j}(x) = (\text{number of } \xi_{ji} \leqq x,\ 1 \leqq i \leqq n_j)/n_j$.

We define $N = n_1/(\sum_{j=1}^{k} n_1/n_j)$ and let $N \to \infty$ mean that $n_j \to \infty$, $j=1, \cdots, k$, so that $n_1/n_j \to \rho_j$, $j=2, \cdots, k$, where ρ_j's are constant for each j.

Under above conditions the following theorems hold.

THEOREM 1.

$$\lim_{N\to\infty} P\left\{N^{1/2} \sup_{a \leqq F(x)} \left(\prod_{j=1}^{k} F_{n_j}(x) - F^k(x)\right) \Big/ F^k(x) < y\right\}$$
$$= (2/\pi)^{1/2} \int_0^{y\{a/(1-a)\}^{1/2}} e^{-t^2/2} dt, \text{ if } y > 0, \text{ zero otherwise, and } 0<a<1.$$

THEOREM 2.

$$\liminf_{N\to\infty} P\left\{N^{1/2} \sup_{a \leqq F(x)} \left|\prod_{j=1}^{k} F_{n_j}(x) - F^k(x)\right| \Big/ F^k(x) < y\right\}$$
$$\geqq 4/\pi \sum_{k=0}^{\infty} (-1)^k (2k+1)^{-1} \exp\{-(2k+1)^2\pi^2(1-a)/8ay^2\},$$
$$\text{if } y > 0, \text{ zero otherwise, and } 0 < a < 1$$
$$= L(y; a).$$

THEOREM 3.

$$\lim_{N\to\infty} P\left\{N^{1/2} \sup_{a \leqq F(x) \leqq b} \left(\prod_{j=1}^{k} F_{n_j}(x) - F^k(x)\right) \Big/ F^k(x) < y\right\}$$
$$= 1/\pi \int_{-\infty}^{y\{b/(1-b)\}^{1/2}} e^{-u^2/2} \cdot \left[\int_0^{\{y[b/(1-b)]^{1/2}-u\}\cdot\{a(1-b)/(b-a)\}^{1/}} e^{-t^2/2} dt\right] du$$

[1] This work was in part supported by the Office of Naval Research, Project ONR 042-023, Contract Nonr-1858(05) to Princeton University.