THE HAUSDORFF DIMENSION OF SINGULAR SETS OF PROPERLY DISCONTINUOUS GROUPS IN N-DIMENSIONAL SPACE

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1. **Hausdorff dimension.** Suppose E is a compact subset of N-dimensional euclidean space, E^N. We denote by $m_\alpha(E)$ the Hausdorff α-dimensional measure of E and by $d(E)$ the Hausdorff dimension of E, i.e. the unique non-negative number such that

$$m_\alpha(E) = 0 \qquad \text{for } \alpha > d(E)$$

and

$$m_\alpha(E) = +\infty \quad \text{for } 0 \leqq \alpha < d(E).$$

We shall need the following result.

THEOREM A [6]. *Let E be a compact subset of E^2. Then $d(E) > 0$ implies E has positive logarithmic capacity.*

2. **Spherical Cantor sets.**

DEFINITION 1 [2], [7]. We say E is a spherical Cantor set if and only if E can be expressed in the form

$$E = \bigcap_{n=1}^{\infty} \bigcup_{i_1, \cdots, i_n = 1}^{K} \Delta_{i_1 \cdots i_n}$$

where K is a positive integer ($K \geqq 2$) and the $\Delta_{i_1 \cdots i_n}$ are closed N-dimensional spheres (of radius $r_{i_1 \cdots i_n}$) satisfying

(a) $\Delta_{i_1 \cdots i_n} \supset \Delta_{i_1 \cdots i_{n+1}}$ $(i_{n+1} = 1, \cdots, K)$,

(b) $\Delta_1, \cdots, \Delta_K$ are mutually disjoint,

(c) there exists a constant A, $1 > A > 0$, such that

$$r_{i_1 \cdots i_n i_{n+1}} \geqq A r_{i_1 \cdots i_n} \qquad (i_{n+1} = 1, \cdots, K)$$

and

(d) there exists a constant B, $1 > B > 0$, such that

$$\rho(\Delta_{i_1 \cdots i_n s}, \Delta_{i_1 \cdots i_n t}) \geqq B r_{i_1 \cdots i_n} \qquad (s, t = 1, \cdots, K; s \neq t)$$

where

$$\rho(S, T) = \inf\{ |s - t| ; s \in S, t \in T\}.$$

We quote the following results.