ON CHERN CLASSES OF REPRESENTATIONS OF FINITE GROUPS

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Let $R(G)$ denote the complex representation ring of a finite group G. Any complex representation ρ of G has invariants $c_n(\rho) \in H^{2n}(G; \boldsymbol{Z})$, the *Chern classes* of ρ (Atiyah [1]).

If H is a subgroup of G, there is the *induced representation* homomorphism

$$i_!: R(H) \to R(G)$$

(cf. [8], say). Atiyah [1] posed the problem of relating the Chern classes of $i_!\lambda$ with those of λ, for any representation λ of H. The purpose of this note is to announce the proof of a conjecture of J. F. Adams which gives some information in this direction; the main idea of the proof was suggested to me by Professor Adams, and is believed to emanate essentially from Professor Atiyah. I would like to thank Professor Adams sincerely for his help, and to acknowledge the helpfulness of Professor Atiyah and Professor M. G. Barratt.

The result to be proved involves the *transfer* homomorphism

$$i_!: H^*(H; \boldsymbol{Z}) \to H^*(G; \boldsymbol{Z})$$

(cf. [6], [8]), and certain linear maps

$$\mathrm{Ch}_k: R(L) \to H^{2k}(L; \boldsymbol{Z})$$

defined, for any finite group L, in terms of the Chern classes as follows:

Let $Q^k(\sigma_1, \cdots, \sigma_n)$ be the polynomial defined by expressing the symmetric polynomial $x_1^k + \cdots + x_n^k$ in indeterminates $x_1, \cdots, x_n$ in terms of the elementary symmetric polynomials $\sigma_i(x_1, \cdots, x_n)$. If $\rho: L \to U(n)$ is a representation of L of degree n, then

$$\mathrm{Ch}_k(\rho) = Q^k(c_1(\rho), \cdots, c_n(\rho)) \in H^{2k}(L; \boldsymbol{Z}).$$

THEOREM 1. *Given any positive integer k, there exists an integer N_k with the following property:*

If H is an arbitrary subgroup of an arbitrary finite group G, then the following diagram of homomorphisms commutes: