FUNCTIONS WITH THE HUYGENS PROPERTY

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A C^2 function $u(x, t)$ belongs to class H, for $a<t<b$, and is called a generalized temperature function, if and only if it is a solution of the generalized heat equation

$$\Delta_x u(x, t) = (\partial/\partial t)\, u(x, t),$$

where $\Delta_x f(x) = f''(x) + (2\nu/x)f'(x)$, ν a fixed positive number. The fundamental solution of this equation is

$$G(x, y; t) = (1/2t)^{\nu+1/2}\mathcal{I}(xy/2t) \exp[-(x^2 + y^2)/4t],$$

with $\mathcal{I}(z) = c_\nu z^{1/2-\nu} I_{\nu-1/2}(z)$, $c_\nu = 2^{\nu-1/2}\Gamma(\nu+\frac{1}{2})$, and $I_\gamma(z)$ the Bessel function of order γ of imaginary argument. We write $G(x; t)$ for $G(x, 0; t)$. The function $u(x, t)$ is said to have the Huygens property, that is, it belongs to class H^*, for $a<t<b$, if and only if $u(x, t) \in H$ there, and

$$u(x, t) = \int_0^\infty G(x, y; t - t')u(y, t')\, d\mu(y), \qquad d\mu(x) = (1/c_\nu)x^{2\nu}\, dx,$$

for every t, t', $a<t'<t<b$, the integral converging absolutely. A generalized heat polynomial $P_{n,\nu}(x, t)$ is defined by

$$P_{n,\nu}(x, t) = \sum_{k=0}^{n} 2^{2k}\binom{n}{k}[\Gamma(\nu + \tfrac{1}{2} + n)/\Gamma(\nu + \tfrac{1}{2} + n - k)]x^{2n-2k}t^k,$$

and its Appell transform $W_{n,\nu}(x, t)$ is given by

$$W_{n,\nu}(x, t) = G(x, t)P_{n,\nu}(x/t, -1/t).$$

The object of this paper is to summarize the principal results derived in characterizing a generalized temperature function which may be represented either by the series expansion $\sum_{n=0}^\infty a_n P_{n,\nu}(x, t)$ or by $\sum_{n=0}^\infty b_n W_{n,\nu}(x, t)$, with convergence taken in the L^2, as well as in the pointwise, sense. Details and proofs will appear later. The work is an extension of the theory developed by Rosenbloom and Widder in [3]. Some of the preliminary results for this study were also de-

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