MATRIX APPLICATIONS OF A QUADRATIC IDENTITY FOR DECOMPOSABLE SYMMETRIZED TENSORS[1]

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1. **Preliminaries.** The Plücker quadratic relations [3, p. 312], [1, p. 52] are examples of necessary and sufficient conditions for a set of

$$\binom{n}{m}$$

numbers to be the coefficients in the expansion of a decomposable skew-symmetric tensor on an orthonormal basis of decomposable skew-symmetric tensors in the symmetry class $\bigwedge^m V$.

The purposes of this announcement are to state necessary relations that obtain among coordinates of decomposable tensors in an arbitrary symmetry class of tensors (Theorem 1) and to indicate (Theorems 2, 3) how these relations can be used to unify and extend a large class of matrix inequalities that includes as special cases the classical Hadamard, Schur, Fan and Fischer results.

In what follows V will denote a fixed n-dimensional unitary space with inner product (x, y). The mth tensor space over V, denoted by $\otimes_{i=1}^m V$, is a unitary space with inner product $(x_1 \otimes \cdots \otimes x_m, y_1 \otimes \cdots \otimes y_m) = \prod_{i=1}^m (x_i, y_i)$. If S_m is the symmetric group of degree m and $\sigma \in S_m$ then $P(\sigma)$ will designate the permutation operator defined on $\otimes_{i=1}^m V$ by

$$P(\sigma)x_1 \otimes \cdots \otimes x_m = x_{\phi(1)} \otimes \cdots \otimes x_{\phi(m)}, \qquad \phi = \sigma^{-1}.$$

If H is a subgroup of S_m of order h and λ is a character on H of degree 1 then

$$T_\lambda = \frac{1}{h} \sum_{\sigma \in H} \lambda(\sigma) P(\sigma)$$

is a hermitian idempotent operator on $\otimes_{i=1}^m V$. Its range is called a symmetry class of tensors and will be denoted by V_λ^m in what follows.

If A is a linear transformation on V then $\Pi^m(A)$ designates the mth Kronecker power of A. The restricted transformation $\Pi^m(A) \mid V_\lambda^m \colon V_\lambda^m \to V_\lambda^m$ is denoted by $K^\lambda(A)$. The map $K^\lambda(A)$ is called an induced mapping, e.g., if $H = S_m$, $\lambda(\sigma) = \epsilon(\sigma)$ then $V_\lambda^m = \bigwedge^m V$ and

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