A GENERALIZATION OF THE HILTON-MILNOR THEOREM

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Communicated by J. Milnor, November 19, 1964

The Hilton-Milnor theorem states that $\Omega\bigvee_{i=1}^{n}\Sigma X_i$ is homotopy equivalent to a weak infinite product, $\prod_{i=1}^{\infty}\Omega\Sigma X_i$, where each X_i, $i>n$, is a smash product of the X_i's, $i\leqq n$. In this note we extend this theorem to the 'wedges' lying between $\bigvee_{i=1}^{n}\Sigma X_i$ and $\prod_{i=1}^{n}\Sigma X_i$.

It will be assumed that all spaces are connected countable CW-complexes with base points. $T_i(X_1, \cdots, X_n)$ is the subset of $X_1\times\cdots\times X_n$ consisting of those points with at least i coordinates at base points. T_0 is the cartesian product and T_{n-1} is the space studied by Hilton and Milnor. T_{n-1} will also be denoted by $\bigvee_{j=1}^{n} X_j$. The smash product $\wedge(X_1, \cdots, X_n)$ is the quotient space $T_0(X_1, \cdots, X_n)/T_1(X_1, \cdots, X_n)$. Define $X^{(n)}$ inductively by $X^{(0)}=S^0$ and $X^{(n)}=\wedge(X^{(n-1)}, X)$, for $n>0$.

The n-fold suspension, $\Sigma^n X$, is defined to be $\wedge(S^n, X)$. The loop space of X, ΩX, is the set of maps, $f\colon I\to X$, such that $f(0)=f(1)=*$. We shall abbreviate $(\Sigma X_1, \cdots, \Sigma X_n)$ and $(\Omega X_1, \cdots, \Omega X_n)$ by $\Sigma(X_1, \cdots, X_n)$ and $\Omega(X_1, \cdots, X_n)$, respectively.

THEOREM 1. *$\Omega T_i\Sigma(X_1, \cdots, X_n)$ is homotopy equivalent to a weak infinite product, $\prod_{j=1}^{\infty}\Omega\Sigma X_j$, where each X_j is equal to $\Sigma^r\wedge(X_1^{(j_1)}, \cdots, X_n^{(j_n)})$ for some $(n+1)$-tuple, $(r, j_1, \cdots, j_n)$, depending upon j. Moreover, the set of $(n+1)$-tuples over which the product is taken is computable.*

If $i=n-1$, Theorem 1 is the Hilton-Milnor theorem. It was proven in [1] by Hilton when the X_i are spheres and extended to the general case by Milnor [2].

We shall sketch the proof of Theorem 1, when $n-i\geqq 2$. The details will appear in [3].

The inclusion map $j\colon T_i(X_1, \cdots, X_n)\to T_0(X_1, \cdots, X_n)$ may be replaced by a homotopy equivalent fibre map, $p\colon E\to T_0$, with fibre F_i. It is easily seen that when $n-i\geqq 2$, the short exact sequence

$$*\to\Omega F_i\to\Omega E\to\Omega T_0\to *$$

splits yielding:

[1] This research was supported in part by National Science Foundation Grant GP-1740.