WEAK TOPOLOGIES ON THE BOUNDED HOLOMORPHIC FUNCTIONS

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Let G be a region in the complex plane such that there is a non-constant bounded holomorphic function on G, and denote the algebra of all such functions by $B_H(G)$. Let $H_\infty(G)$ denote the Banach algebra that arises when $B_H(G)$ is endowed with the supremum norm. In the case where G is the unit disc D, $H_\infty(G)$ has been extensively studied, mostly by a real-variables analysis of the radial boundary values of bounded holomorphic functions.

We consider here another natural topology on $B_H(G)$, namely, the strict topology β, introduced by Buck in [2], where he made a preliminary study of the case $G=D$. In the work that we outline in this announcement, a number of intrinsic complex-variables methods are introduced, and used in conjunction with the theory of topological linear spaces to investigate the structure of $\beta(G)$, and to shed some light also on the structure of $H_\infty(G)$. Some of the general results obtained here were obtained in a different form, in the special case $G=D$, by Brown, Shields, and Zeller [1].

One of our results is that $H_\infty(G)$ is always the dual of a certain Banach space of equivalence classes of measures. Letting $\alpha(G)$ denote $B_H(G)$ under the weak topology arising from this duality, we exhibit strong similarities between $\alpha(G)$ and $\beta(G)$ that can be exploited in studying the ideal structure of $\beta(G)$.

The main new tool is the balayage, or sweeping, of measures. In point of fact, several different methods of balayage are used. One is similar to the balayage of potential theory, another uses duality of certain topological vector spaces, and the third uses ideas related to the Cauchy integral formula.

We present here only our main results, without proof. The full details will be published elsewhere. The order in which the results are presented is a little artificial, since some of the structure theorems depend on the balayage theorems.

Structure.

DEFINITION. *$\beta(G)$ is $B_H(G)$ under the topology given by the seminorms*

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