RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited. Manuscripts more than eight typewritten double spaced pages long will not be considered as acceptable.

PRINCIPAL FUNCTIONS FOR ELLIPTIC SYSTEMS OF DIFFERENTIAL EQUATIONS[1]

BY FELIX E. BROWDER

Communicated October 26, 1964

Introduction. Let A be an elliptic system of linear differential operators on an open set V of R^n (or, more generally, an elliptic differential operator in a vector bundle B over a manifold V). If V_1 is an open subset of V, the principal function problem for A is intuitively the following: Given a solution s of $As=0$ in V_1, to find a solution p of $Ap=0$ on all of V such that, on V_1, $u=p-s$ is a "nice" solution of $Au=0$, i.e., u satisfies prescribed boundedness and boundary conditions.

For the case of a single self-adjoint second-order elliptic operator, principal functions were introduced by L. Sario and studied systematically by him and his collaborators in [1]–[6] by making strong use of the maximum principle and the Harnack inequality. It is our object in the present paper to indicate an extremely direct and simple proof of the existence of principal functions for the general class of linear elliptic systems of differential operators.

1. Let V_1 be an open subset of V. We consider r-vector functions $u=(u_1, \cdots, u_r)$ on V; let $x=(x_1, \cdots, x_n)$ be the general point of V, and let $C_c^\infty(V)$ be the family of infinitely differentiable functions u with compact support in V. We shall consider elliptic systems A of the form

$$Au = \sum_{|\alpha|\leq m} A_\alpha(x) D^\alpha,$$

where for each n-tuple $\alpha=(\alpha_1, \cdots, \alpha_n)$ of non-negative integers, D^α is the elementary differential operator $\prod_{j=1}^n (\partial/\partial x_j)^{\alpha_j}$ and A_α is an $(r\times r)$-matrix function on V. For simplicity, we assume that A_α is infinitely differentiable to avoid complication of statement though all statements are valid under very mild regularity conditions.

[1] The preparation of this paper was partially supported by NSF Grant GP-3552. The writer is indebted to L. Sario for stimulating his interest in this problem.