

## ON NEW RESULTS IN THE THEORY OF MINIMAL SURFACES

JOHANNES C. C. NITSCHÉ<sup>1</sup>

### CHAPTER I. INTRODUCTION

It was with some hesitation that I decided to talk about the theory of minimal surfaces. Although, over the centuries, minimal surfaces and the problems connected with them have attracted the never-ending interest of mathematicians, owing to their classical character and the fact that the theory, without doubt, reached its culmination in the 1930's,<sup>2</sup> marked by the spectacular and pioneering achievements of L. Tonelli, R. Garnier, T. Radó, J. Douglas, R. Courant, C. B. Morrey, E. M. McShane, M. Shiffman, M. Morse, T. Tompkins and others, one might expect to find a closed chapter. This is not the state of affairs, however.

New and important results have been forthcoming ever since—so the results of M. Shiffman [3] on doubly-connected minimal surfaces, the studies of L. Bers [1], [2], [3] about Abelian minimal surfaces and about boundary-value problems for the minimal surface equation and isolated singularities of its solutions, the theorems of H. Lewy [5], [6] on the boundary behavior of minimal surfaces, the papers of E. J. Mickle [2], E. Hopf [1], [2], E. Heinz [1], K. Jörgens [1], R. Osserman [1], [3], [4] on Bernstein's theorem and its generalizations, of Y. W. Chen [1], [2], [3] on new versions of Plateau's problem and on branch points, the ideas of R. Finn [1], [2], [3], [6], [7] concerning the minimal surface equation and its generalizations, the publications of J. Serrin [2], H. Jenkins [1], [2], and H. Jenkins and J. Serrin [1] on a priori estimates for the minimal surface equation and more general variational problems, the work of R. Osserman [5]–[8] on complete minimal surfaces, studies of the relations between minimal surfaces in isothermic representation and function theory by M. Tsuji [1], M. O. Reade [1], E. F. Beckenbach and G. A. Hutchison [1], and many others, the manifold investiga-

---

An address delivered before the Chicago meeting of the Society on April 25, 1964, by invitation of the Committee to Select Hour Speakers for Western Sectional Meetings; received by the editor, August 28, 1964.

<sup>1</sup> This work was supported by Contract Nonr 710 (54) between the University of Minnesota and the Office of Naval Research.

<sup>2</sup> It might be of interest to note the following comment of two geometers (see W. Blaschke and H. Reichardt [1, p. 118]): "If one brings into comparison the first and the last of these works (i.e. the surveys by E. Beltrami [1] and T. Radó [6]) one can confront the stormy youth of a geometric question with its tired old age."