THE KERVAIRE INVARIANT OF $(8k+2)$-MANIFOLDS

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1. Statements of results. Let $\Omega_m(\mathrm{Spin})$, $\Omega_m(SU)$, and $\Omega_m(e)$ denote the mth spinor, special unitary, and framed cobordism groups respectively (see [6]). In [4] Kervaire defined a homomorphism $\Phi\colon \Omega_{2n}(e) \to Z_2$ for n odd and $n \neq 1$, 3, or 7, and showed that $\Phi=0$ for $n=5$. Kervaire and Milnor state in [5] that $\Phi=0$ for $n=9$. One of the corollaries of our results is that $\Phi=0$ for $n=4k+1$, $k\geqq 1$.

In [2] a homomorphism $\Psi\colon \Phi_{2n}(\mathrm{Spin}) \to Z_2$ was defined for $2n=8k+2$, $k\geqq 1$, such that $\Phi=\Psi\rho$, where $\rho\colon \Omega_{2n}(e)\to\Omega_{2n}(\mathrm{Spin})$ is the obvious map. Ψ induces a map from $\Omega_{2n}(SU)$ into Z_2 which we also denote by Ψ. It is easily verified that $\Omega_1(SU)=\Omega_1(\mathrm{Spin})=\Omega_1(e)=Z_2$. Let α be the generator. Let θ be the secondary cohomology operation coming from the relation $Sq^2Sq^2=0$ on an integer cohomology class [7]. If f is a map, let θ_f denote the associated functional cohomology operation [7].

The main theorems of this announcement are the following.

THEOREM 1.1. *If* $\beta\in\Omega_{8k}(\mathrm{Spin})$ *and* $k\geqq 1$, *then* $\Psi(\alpha^2\cdot\beta)=\chi(\beta)$, *where* $\chi(\beta)$ *is the Euler characteristic of* β *reduced* mod 2.

THEOREM 1.2. *If* $\beta\in\Omega_{8k}(SU)$ *and* $k\geqq 1$, *then* $\Psi(\alpha^2\cdot\beta)=\theta_\nu(v^2)(M)$, *where* M *is a 3-connected manifold representing* $\alpha^2\cdot\beta$, $\nu\colon M\to BSU$ *is the classifying map of the SU-structure on the normal bundle of* M, *and* $v\in H^{4k}(BSU;\, Z)$ *is such that* v *reduced* mod 2 *is* v_{4k} *in the expression* $\overline{W}=Sq(1+v_2+v_4+\cdots)$.

We now deduce some corollaries of these two theorems.

COROLLARY 1.3. $\Phi\colon \Omega_{8k+2}(e)\to Z_2$ *is zero if* $k\geqq 1$.

PROOF. Conner and Floyd [9] and Lashof and Rothenberg[2] have shown that if $\gamma\in\Omega_{8k+2}(SU)$ goes to zero in $\Omega_{8k+2}(U)$, then $\gamma=\alpha^2\cdot\beta$, where $\beta\in\Omega_{8k}(SU)$. In particular, if $\gamma=\bar{\rho}(\delta)$, $\delta\in\Omega_{8k+2}(e)$, then $\gamma=\alpha^2\cdot\beta$. Let $\delta=[M]$. Then M can be taken to be 3-connected and ν is homotopic to a constant. The corollary now follows from Theorem 1.2.

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