spread snobbism to the contrary, correct mathematics is a proper tool for obtaining physically relevant results.

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Probabilities on algebraic structures. By Ulf Grenander. Wiley, New York, 1963. 218 pp. $12.00.

The classical limit theorems of probability theory exemplify what may be termed the "large-number phenomenon." Stated roughly it is this: in combining a large number of independent random variables subject to certain "mild" restrictions, the outcome will be asymptotically either a well-determined number, or a random variable with a well-determined distribution. For example, in one version of the central limit theorem, we form $f(x_1, \cdots, x_n) = n^{-1/2}(x_1 + \cdots + x_n)$, where the x_i are independent random variables with distribution functions F_i subject only to the restrictions:

$$\int |x|^3 dF_i(x) < M < \infty, \quad \int x^2 dF_i(x) = \sigma^2, \quad \int x\, dF_i(x) = 0.$$

The conclusion is that $f(x_1, \cdots, x_n)$ is asymptotically a normal random variable with mean 0 and variance σ^2.

The variables $x_1, \cdots, x_n$ need not be combined linearly. The following instance of the large-number phenomenon, which illustrates this, has recently received attention for its possible application to nuclear physics. Suppose $x_{11}, \cdots, x_{nn}$ are n^2 random entries in a large symmetric matrix with eigenvalues $\lambda_1, \cdots, \lambda_n$. Let

$$f_u(x_{11}, \cdots, x_{nn})$$

denote the proportions of the eigenvalues which do not exceed $u \max_{1 \leq i \leq n} \lambda_i$. Imposing mild restrictions on the variables $x_{11}, \cdots, x_{nn}$, we find that the functions f_u, which are highly complex functions of the variables, tend to well-determined values as $n \to \infty$. (See Chapter 7 of the book under review for more details.)

Clearly, the elucidation of the scope of this phenomenon is a major problem for probabilists. One of the motivations for studying "probabilities on algebraic structures" is that it provides a systematic approach to this problem. Namely, the instances of the large-number phenomenon that classical probability theory discovered involved addition of real-valued random variables. It is reasonable to expect that by copying its methods one can extend these results to random variables taking values in more general algebraic structures and being combined in accordance with the relevant laws of composition.