A TRANSCENDENTAL VIEW OF THE SPACE OF ALGEBRAIC RIEMANN SURFACES

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The subject of my lecture is not a large-scale and general theory but rather a special problem in conformal mapping. By way of justification for bringing it to the attention of a broader audience I may cite, first of all, its deep nature and honorable origin—the problem originated in Riemann's memoir [46] of 1857 on algebraic functions, and after the passage of a century some definitive insight into it is only now emerging as a result of the collective efforts of a group of mathematicians. Secondly, it is intimately bound up with a variety of topics of more general interest: conformal mapping with particular reference to extremal problems, several complex variables, and partial differential equations, on the one hand; topology and algebraic geometry, on the other and, in particular aspects, even group representations and arithmetic.

1. **Introduction and history.** Briefly put, and in modern paraphrase, Riemann observed: (a) that for compact (algebraic) Riemann surfaces of genus $g=0$, i.e., simply connected, one has the analogue of the Riemann mapping theorem, that is, all such surfaces not only admit *topological* maps on one another but also *conformal* maps, in particular, on the Riemann sphere (all maps of surfaces in this paper are assumed to be homeomorphisms without further mention); (b) for $g>0$ it is no longer true that the existence of a map of one surface on another (equivalent to their having the same genus) implies the existence of a conformal map; and (c) as a partial compensation for (b) and substitute for (a) in case $g>0$, if one considers the *conformal equivalence classes* obtained by identifying surfaces which admit mutual conformal maps, these classes form not an isolated point as in (a) but in some sense or other a *continuum* of *finite* dimension, in fact, of dimension one for $g=1$ and $3g-3$ for $g\geqq 2$. Riemann called a set of parameters for the continuum *moduli* (Riemann and his immediate successors often spoke cryptically of *the* moduli, as though some particular set of moduli were implicitly distinguished—this has not been borne out by the modern research to be described below).

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