# AN EXPLICIT INVERSION FORMULA FOR FINITE-SECTION WIENER-HOPF OPERATORS[1]

BY GLEN BAXTER AND I. I. HIRSCHMAN, JR.

Communicated by A. Zygmund, July 14, 1964

Let $T$ be the real numbers modulo 1 and $\mathcal{A}_0$ the algebra of complex continuous functions $f(\theta)$ on $T$ which have absolutely convergent Fourier series. For $f(\theta) \in \mathcal{A}_0$ we set

$$\|f\|_0 = \sum_{-\infty}^{\infty} | f(k) |$$

where

$$f(k) = \int_T f(\theta) e^{-2\pi i k\theta} \, d\theta.$$

For $f \in \mathcal{A}_0$ we define

$$E^+(n)f \cdot (\theta) = \sum_{k \geqq n} f(k) e^{2\pi i k\theta},$$

$$E^-(n)f \cdot (\theta) = \sum_{k \leqq n} f(k) e^{2\pi i k\theta}.$$

DEFINITION. Let $\mathcal{A}$ be a Banach algebra of complex continuous functions $f(\theta)$ on $T$ with norm $\|\cdot\|$. $\mathcal{A}$ will be said to be of type $\mathfrak{M}$ if the following conditions are satisfied:

1. $\mathcal{A}_0 \supset \mathcal{A}$, $\|f\|_0 \leqq \|f\|$ for all $f \in \mathcal{A}$;
2. $e^{2\pi i k\theta} \in \mathcal{A}$ for $k = 0, \pm 1, \pm 2, \cdots$, and the trigonometric polynomials are dense in $\mathcal{A}$;
3. there exists a constant $M$ independent of $n$ such that

$$\|E^+(n)f\| \leqq M\|f\|, \qquad \|E^-(n)f\| \leqq M\|f\|, \qquad \text{all } f \in \mathcal{A}.$$

For $c \in \mathcal{A}$ we define the finite-section Wiener-Hopf operators

$$W_c^+(n)f = E^+(0)E^-(n)cE^+(0)E^-(n)f,$$

$$W_c^-(n)f = E^-(0)E^+(-n)cE^-(0)E^+(-n)f.$$

Here $n \geqq 0$. Our principal result is the identities below. These identities are algebraic in character and can be seen to hold in a much more general (even in a noncommutative) setting than that considered here. We have preferred to present them in a context requiring as few definitions and as little machinery as possible.

[1] This research was supported in part by the United States Air Force through the Air Force Office of Scientific Research and Development Command under Contract No. AF-AFOSR 63-381.