COBORDISM CLASSES OF SQUARES OF ORIENTABLE MANIFOLDS

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In this paper we give an outline of the following theorem.[1] Full details will appear elsewhere.

THEOREM. *If M is an orientable manifold, then there exists a spin manifold N such that N is cobordant to $M \times M$ (in the unoriented sense). (For definitions and notation see* [1] *and* [3].)

Following C. T. C. Wall [5] we construct a set of orientable manifolds whose cobordism classes generate the image of the "orientation ignoring homomorphism" $r: \Omega \rightarrow \mathfrak{N}$, and the theorem is then verified for each of these generators.

Some of these manifolds are certain complex projective spaces CP^n. As was noted in [2], $CP^n \times CP^n$ is cobordant to quaternionic projective space HP^n. Since HP^n is always 3-connected it is a spin manifold.

A second type of manifold used is constructed as follows. Let λ be the canonical nontrivial line bundle over real projective space P^n, and ϵ^m the trivial m-plane bundle over P^n. Define $M(m, n)$ as the space of lines through the origin in each fibre of the Whitney-sum bundle $\lambda \oplus \epsilon^n$. $M(m, n)$ is an orientable manifold if and only if m is odd and n is even, and certain of these manifolds are used as generators for $r(\Omega)$.

The third type of manifold used is denoted by

$$M(m_1, n_1; m_2, n_2; \cdots ; m_{r+1}, n_{r+1}),$$

where $r \geqq 1$, m_i is odd and n_i is even for $i = 1, \cdots, r+1$. This manifold is the total space of a certain fibre bundle over $S^1 \times \cdots \times S^1$ (r factors), with fibre $M(m_1, n_1) \times \cdots \times M(m_{r+1}, n_{r+1})$.

To prove the theorem for these last two types of manifolds we construct their "complex analogues" as follows. Let $c\lambda$ denote the canonical complex line-bundle over complex projective space CP^n, and $c\epsilon^m$ the trivial complex m-plane bundle over CP^n. Then $CM(m, n)$ is the space of complex lines through the origin in each fibre of $c\lambda \oplus c\epsilon^m$. $CM(m_1, n_1; \cdots ; m_{r+1}, n_{r+1})$ will be the total space of a fibre

[1] This theorem was originally conjectured by J. Milnor in [2].