SOME NEW RESULTS IN DEFINABILITY

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Consider a first-order language $\mathcal{L}$ with identity and finitary predicate symbols. We let the letter A range over the models for $\mathcal{L}$. Let $\mathbf{P}$, $\mathbf{Q}$ be new unary predicate symbols and let $\mathcal{L}(\mathbf{P}, \mathbf{Q})$ be the new first-order language with the additional predicates $\mathbf{P}$ and $\mathbf{Q}$. Models for $\mathcal{L}(\mathbf{P}, \mathbf{Q})$ will be written as (A, P, Q) where P and Q are subsets of A. Let T and S be sets of sentences of $\mathcal{L}(\mathbf{P}, \mathbf{Q})$ (or of $\mathcal{L}(\mathbf{P})$, or $\mathcal{L}$). We write $T \vdash S$ to mean that every model of T is a model of S. $|X|$ shall denote the cardinal of the set X.

The following two known results are due to Beth [1] and Svenonius [4].

(I) BETH'S THEOREM. *Let T be a theory in $\mathcal{L}(\mathbf{P})$. Then the following are equivalent.*

(i) *There exists a formula $F(t)$ of $\mathcal{L}$ such that*

$$T \vdash \forall t(\mathbf{P}(t) \leftrightarrow F(t)).$$

(ii) *For every model A for $\mathcal{L}$, the set*

$$X_A = \{P \mid (A, P) \text{ is a model of } T\}$$

has at most one element.

(II) SVENONIUS' THEOREM. *Let T be a theory in $\mathcal{L}(\mathbf{P})$. Then the following are equivalent.*

(i) *There exists a finite number of formulas $F_1(t), \cdots, F_n(t)$ of $\mathcal{L}$ such that*

$$T \vdash \bigvee_{1 \leq i \leq n} \forall t(\mathbf{P}(t) \leftrightarrow F_i(t)).$$

(ii) *For every model (A, P) of T, the set*

$$X_{A,P} = \{P' \mid (A, P') \cong (A, P)\}$$

has exactly one element P.

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