AN APPROACH TO QUASILINEAR ELLIPTIC PROBLEMS

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Communicated by A. E. Taylor, April 23, 1964

1. **Introduction.** Some of the common methods used to prove existence theorems for quasilinear elliptic equations employ the Schauder fixed-point theorem, the Schauder-Leray theory, or some variant of these in a Banach space [1], [3], [4]. It is also usual to consider the principal part of the equation to be in divergence form, when strong ellipticity and the Dirichlet problem are natural concepts [1], [5], [11]. This leaves the question of regularity of the solution to be dealt with separately. With the wealth of existence and regularity theory now available for general linear elliptic equations it seems desirable to extend this work as directly as possible to quasilinear elliptic equations with continuous coefficients but without assuming divergence form. This announcement indicates some results on this approach. In all applications we rely on known linear existence and regularity theory. Unavoidably the technique requires the existence of an a priori estimate, but very naive estimates can be made to yield results easily. The results so obtained are usually not the most general ones known in specific cases. The central ideas are presented in the next section as two theorems which can be both generalized and specialized considerably as abstract theorems. The form we have given however, is that most useful in applications to quasilinear elliptic equations. Some simple examples of the use of these theorems are given in the last section. Proofs and the application to general elliptic and parabolic equations will appear elsewhere.

2. **Existence theorems.** Let X, Y be Banach spaces with norms denoted by $|\cdot|$, in both spaces, and denote by $[X, Y]$ the Banach space of continuous linear operators on X into Y with the uniform topology. Let θ: $u\to\theta(u)$ be a not necessarily linear mapping from X into $[X, Y]$ with the following properties: there exists a closed convex subset U of X and a subset W of Y such that

(2.1) the restriction of θ to U is compact, i.e., θ maps a bounded sequence of elements of U into a sequence of elements of $[X, Y]$ containing a convergent subsequence,

(2.2) the restriction of θ to U is continuous, i.e., if $\{u_n\}$ is a sequence of elements of U, such that $u_n\to u\in U$ as $n\to\infty$ then $\theta(u_n)\to\theta(u)$ in $[X, Y]$ as $n\to\infty$,

[1] The research and preparation of this paper was supported (in part) by the National Science Foundation grants GP-2045 and GP-1988.