INJECTIVE ENVELOPES OF BANACH SPACES ARE RIGIDLY ATTACHED

BY J. R. ISBELL[1]

Communicated by V. Klee, May 18, 1964

Introduction. H. B. Cohen has constructed [2] an injective (linear) envelope $\epsilon_l E$ for every real or complex Banach space E, and shown that it is unique up to a linear isometry. The real case of Cohen's result provides most of the answer to a question I had asked, which concerned the injective metric envelope $\epsilon_m E$. It is known that in real Banach spaces the metric and metric linear notions of "injective" coincide [1], [6]. The question whether the metric space $\epsilon_m E$ and the Banach space $\epsilon_l E$ coincide is unambiguous, at least in the sense that the metric structure of a real Banach space determines the linear structure [5]. The answer:

THEOREM 1. *For a real Banach space E, $\epsilon_l E = \epsilon_m E$.*

I cannot deduce this from Cohen's results on ϵ_l, but I get it from my results on ϵ_m [4]. Either approach yields the real case of

THEOREM 2. *A linear autoisometry of a Banach space E can be extended in only one way to a linear autoisometry of $\epsilon_l E$.*

However, the proof in the manner of Cohen covers both cases and is shorter.

Each approach proves a strengthened form of Theorem 2. By a lemma of Cohen [2], any other linear extension has norm >1. In the real case, any nonlinear extension increases some distance.

Proofs. Let us do Theorem 2 first. It suffices to show that the subspace of all points of $\epsilon_l E$ left fixed by a linear autoisometry T different from the identity lies in an injective proper subspace. Now the form of $\epsilon_l E$ is known (Nachbin-Goodner-Kelley-Hasumi Theorem, re-proved in [2]); it is the space $C(X)$ of all continuous scalar-valued functions on an arbitrary extremally disconnected compact space. The form of T is easy to determine; it must consist of composition with an autohomeomorphism τ of X and multiplication by a continuous function t on X to the scalars of absolute value 1. (This is readily deduced from the characterization of the extreme points of $C(X)^*$ [3].)

Since $T \neq 1$, either $\tau \neq 1$ or $\tau = 1$ but $t \neq 1$. In the former case there

[1] Supported by the National Science Foundation.