# TAMING CANTOR SETS IN $E^n$

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1. **Introduction.** Any two compact, perfect, zero-dimensional and nondegenerate metric spaces are homeomorphic. We call such a space a *Cantor set.* A Cantor set $C$ in Euclidean space $E^n$ is called *tame* if there is a homeomorphism $h$ of $E^n$ onto $E^n$ such that $h(C) \subset E^1 \times \{0_{n-1}\} = E^1 \subset E^n$. For examples of *wild* (i.e., nontame) Cantor sets, see [1], [4], [3], and [9]. The examples of Blankenship [3] give the existence of wild Cantor sets in $E^n$ for each $n \geqq 3$.

Homma [8] and Bing [2, Theorem 5.1] have shown that a Cantor set $C$ in $E^3$ is tame if and only if $E^3 - C$ is 1-ULC (definition below). It is our purpose here to extend this useful characterization to Cantor sets in $E^n$ $(n \neq 4)$. We assume the customary metric on $E^n$ throughout this paper. Let $K$ be a compact set in $E^n$. Then we say that $E^n - K$ is 1-ULC if for each $\epsilon > 0$ there is a $\delta > 0$ such that each loop (i.e., closed curve) of diameter less than $\delta$ in $E^n - K$ is null-homotopic in $E^n - K$ on a set of diameter less than $\epsilon$.

We sketch the proof below, relying heavily on the *cellularity criterion* [10, Theorems 1 and 1′]. For $n \geqq 5$, this criterion implies that a compact absolute retract $X$ in the interior of a piecewise-linear (abbreviated pwl) $n$-manifold $M^n$ is cellular with respect to piecewise-linear cells if and only if for each open set $U \subset M$ containing $X$ there is an open set $V$ such that $X \subset V \subset U$ and each loop in $V - X$ is null-homotopic in $U - X$.

2. **The theorem.** We first state some lemmas. For Lemma 1, see [11, Theorem 3], [12, Theorem 4], and [6, Theorem 3]. In Whitehead's theorem [12], we take $K = \text{Bd } M$.

LEMMA 1. *Let $M^n$ be a compact piecewise-linear n-manifold (possibly with boundary), and let $E_1$ and $E_2$ be piecewise-linear n-cells in* Int $M$. *Then there is a piecewise-linear homeomorphism $h$: $M \to M$ such that $h(E_1) = E_2$ and $h|$*Bd $M$ *= the identity.*

LEMMA 2. *Let $C$ be a Cantor set in $E^n$, $n \geqq 3$. Then $C$ is tame if for each $\epsilon > 0$ there is a finite, disjoint collection of piecewise-linear n-cells, each of diameter less than $\epsilon$, whose interiors cover $C$.*

The proof of Lemma 2 is essentially the same as in the three-di-

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