NONLOCAL ELLIPTIC BOUNDARY VALUE PROBLEMS

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1. **Introduction.** Let A be an elliptic operator on a region G. Boundary value problems of the form $Au=f$ on G, $Bu=0$ on the boundary ∂G, have been studied extensively for the case in which B is a system of differential operators; see, for example, [1], [4], [5], [9]. Bade and Freeman [3] have obtained results for a class of nonlocal problems, i.e., problems in which the operators B are not necessarily differential operators. In [3], A is taken to be the Laplace operator and B is of the form $Bu=\partial u/\partial n-Cu$, where $\partial/\partial n$ is the normal derivative and C is any bounded operator in $L^2(\partial G)$.

In this note we indicate some extensions of the results of [3] to general classes of nonlocal boundary value problems for elliptic operators of arbitrary even order with variable coefficients. Details and proofs will appear elsewhere. This research is part of the author's doctoral dissertation, prepared under the direction of Professor Felix Browder at Yale University. The author is grateful to Professor Browder for his advice and encouragement.

2. **Main results.** For the definition of various spaces of functions and distributions we refer to [7, §1]. The operators considered are all linear, and we denote the domain and range of an operator T by $D(T)$ and $R(T)$, respectively.

Let A be an elliptic operator of order $2p$ defined on a region $G\subseteq E^n$, and let $(B_0, B_1, \cdots, B_{2p-1})$ be a system of differential operators defined near the boundary ∂G. The boundary value problems to be considered here are of the form

$$(*)\qquad Au=f, \qquad B_k u = \sum_{j\in J} C_{kj}B_j u, \qquad k\in K,$$

where (C_{kj}) is a system of operators defined in suitable function spaces on ∂G and the index sets J and K are complementary subsets of $\{0, 1, \cdots, 2p-1\}$.

Let A_1 be the maximal operator for A, i.e., the operator in $L^2(G)$ which is the restriction of A to those functions u in $L^2(G)$ such that the distribution Au is in $L^2(G)$. We wish to study the restriction of A_1 to those functions satisfying in some sense the above boundary conditions. This can be accomplished by studying perturbations of

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