INCIDENCE MATRICES WITH THE CONSECUTIVE 1's PROPERTY

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1. **Introduction.** Let $A=(a_{ij})$ be an m-by-n matrix whose entries a_{ij} are all either 0 or 1. For certain applications, one of which will be discussed below, it is of interest to know whether there is an m-by-m permutation matrix P such that the 1's in each column of PA occur in consecutive positions. In this note we state certain results that have relevance for this problem. Proofs of these, together with an efficient computational method for deciding the question in any given case, will be published elsewhere.

The problem posed above includes that of determining whether a given finite undirected graph is an interval graph. The study of interval graphs [2], [3], [4], [5] was stimulated in part by an application concerning the fine structure of genes. A basic genetic problem, discussed in [1], is to decide whether or not the sub-elements of genes are linked together in a linear order. A way of approaching this problem is also described in [1]. Briefly, it is as follows. For certain microorganisms, there are a standard form and mutants, the latter arising from the standard form by alteration of some connected portion of the genetic structure. Experiments can be devised for determining whether or not the blemished parts of two mutant genes intersect or not. Thus the mathematical problem becomes: Given a large number of mutants together with intersection data on pairs of mutants, to decide whether this information is compatible with a linear model of the gene. If one represents the intersection data by a graph (two mutants, i.e., vertices, being joined by an edge if their blemished portions intersect), the problem is to decide whether this graph is an interval graph.

2. **A basic theorem.** We say that a (0, 1)-matrix A has the *consecutive 1's property* (for columns) if there is a permutation matrix P such that the 1's in each column of PA occur consecutively. The first question that naturally arises is how much information about A is needed to decide whether it has the property or not. Do we need to

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