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## A NOTE ON APPROXIMATION BY BERNSTEIN POLYNOMIALS

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Let  $f$  be continuous on  $[0, 1]$  and  $0 \leq \alpha < \beta \leq 1$  and let  $B_n f$  be the Bernstein polynomial of  $f$  of degree  $n$ , defined by

$$B_n f(x) = \sum_{\nu=0}^n f\left(\frac{\nu}{n}\right) \binom{n}{\nu} x^\nu (1-x)^{n-\nu}.$$

In view of a result of E. V. Voronovskaya, which states that the boundedness of  $f$  on  $[0, 1]$  and the existence of  $f''$  at a point  $x \in [0, 1]$  implies that

$$B_n f(x) - f(x) = \frac{x(1-x)}{2n} f''(x) + o\left(\frac{1}{n}\right) \quad (n \rightarrow \infty),$$

it has been conjectured [1, p. 22] that the relation

$$B_n f(x) - f(x) = o\left(\frac{1}{n}\right)$$

cannot be true for all  $x \in [\alpha, \beta]$  unless  $f$  is a linear function on  $[\alpha, \beta]$ . The following theorem related to this conjecture was proved by K. de Leeuw [2]:

If  $f$  is continuous on  $[0, 1]$  and

$$B_n f(x) - f(x) = O\left(\frac{1}{n}\right)$$