BOOK REVIEWS

Flows on homogeneous spaces. By L. Auslander, L. Green, and F. Hahn, with the assistance of L. Markus and W. Massey and an Appendix by L. Greenberg. Annals of Mathematics Studies Number 53. Princeton University Press, Princeton, N. J., 1963. 7+107 pp. $2.75.

This highly significant book should be pleasing to a wide variety of mathematical tastes. It is a study by diverse methods of the topological and measure-theoretic behavior of flows associated with Lie groups. The contents should probably be described neither as general theorems nor special examples, but either as relatively special theorems or relatively general examples.

A one-parameter subgroup of a Lie group G acts on a homogeneous space G/D of G by left multiplication to produce a "coset" flow, a flow being a transformation group determined by the real line acting upon a phase space which in this case is G/D. This collection of closely related papers forms an extended investigation of coset flows, where G is usually assumed to be nilpotent or, more generally, solvable, with respect to properties centrally located in topological dynamics and ergodic theory. The techniques which the authors use range over Lie groups, Lie algebras, general topology, homology theory, fiber bundles, differential geometry, measure theory, functional analysis, spectral theory, and infinite-dimensional group representations, with a bonus appearing in connection with diophantine approximation. An impressive exhibition of breadth and depth in mathematical knowledge and skill, the book is a record of some of the research work done under the auspices of the NSF-sponsored conference "Analysis in the Large" at Yale University during the academic year 1960–61.

Chapters I and II are largely summaries of known theories and are intended to smooth the reader's path into the complicated considerations of the ensuing parts of the book. The chapters and appendices after Chapter II are research papers which contain many new and interesting results. The contents of these one hundred closely packed pages can only be cursorily and very inadequately indicated here by at most a few sentences for each chapter and appendix. Any particular theorems quoted are samples only.

For the sake of capsule exposition let us agree on the following terminology, which is mostly the usage in the book. If G is a connected simply-connected noncompact Lie group, if D is a discrete