A STRICT MAXIMUM THEOREM FOR ONE-PART FUNCTION SPACES AND ALGEBRAS

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The idea of a "part" of the spectrum of a function algebra was introduced by Gleason in [3]. Recently Bishop [2] extended one of the results of [3] and showed that any two points in one part of a function algebra have representing measures which are mutually absolutely continuous. Motivated by Bishop's work, we have shown [1] that the concept of a part extends to arbitrary linear spaces of functions in a way which generalizes the idea for an algebra. Our purpose here is to show how this concept can be used to prove a strict maximum theorem for subalgebras or subspaces.

Let A be a separating, uniformly closed subalgebra of $C_c(X)$, all continuous complex-valued functions on a compact space X, and let S_A denote the space of nonzero homomorphisms of A. Then S_A is a compact subset of A^* with the w^* topology, and X is homeomorphically embedded in S_A as the set of evaluation functionals. The functions in A are generally regarded as being extended from X to continuous functions on all of S_A. If A^* is given the norm topology rather than the customary w^* topology, then the relation $x \sim y$ ($x, y \in S_A$) defined by $\|x-y\|<2$ is an equivalence relation on S_A, and the equivalence classes are called the Gleason parts of S_A.

Now let $C_r(X)$ be all continuous real functions on X, and let B be a separating subspace of $C_r(X)$ which contains the constants. The parts of X (with respect to B) are defined to be the equivalence classes under the relation $x \sim y$ defined by the condition that there exist a real number $a>1$ such that $1/a<u(x)/u(y)<a$ for all positive functions $u \in B$. In [1] it is shown that if $B = \mathrm{Re}\, A$ for a function algebra A, then the B-parts coincide with the A-parts. We obtain a geometric interpretation of a part by regarding X as embedded in B^*, and representing B as the weak dual of B^*, restricted to the closed convex hull T_B of X in B^*. The parts of T_B, regarding B as a subspace of $C_r(T_B)$, are (see [1]) those (necessarily convex) subsets Π of T_B which do not contain any extreme points of T_B, and have the property that if $x, y \in \Pi$, then the line in B^* joining x and y intersects T_B in a segment containing x and y in its interior. The parts of

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