THE RECURSIVE EQUIVALENCE TYPE OF A CLASS OF SETS[1]

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1. **Introduction.** Let us consider non-negative integers (*numbers*), collections of numbers (*sets*) and collections of sets (*classes*). The letters ϵ and o stand for the set of all numbers and the empty set of numbers respectively. We write $\subset$ for inclusion, proper or improper. A mapping from a subset of ϵ into ϵ is called a *function*; if f is a function, we denote its domain and its range by δf and ρf respectively. Let a class of mutually disjoint nonempty sets be called an *md-class*; such a class is therefore countable, i.e., finite or denumerable. We recall that the *recursive equivalence type* (abbreviated: RET) of a *set* α, denoted by $\mathrm{Req}(\alpha)$, is defined [1, p. 69] as the class of all sets which are recursively equivalent to α. We wish to consider the problem: "*How can we define the RET of an md-class in a natural manner?*" Throughout this note S stands for an md-class and σ for the union of all sets in S; for every $x\in\sigma$ we denote the unique set α such that $x\in\alpha\in S$ by α_x.

DEFINITIONS. A set γ is a *choice set* of S, if

(1) $\gamma\subset\sigma$,

(2) γ has exactly one element in common with each set in S.

The set γ is a *good* choice set of S (abbreviated: gc-set), if it also satisfies

(3) there exists a partial recursive function $p(x)$ such that $\sigma\subset\delta p$ and $(\forall x)[x\in\sigma\Rightarrow p(x)\in\gamma\cdot\alpha_x]$.

Consider the special case that the md-class S is a finite class of finite sets. Then

(a) every choice set of S is a good choice set,

(b) every two choice sets of S are recursively equivalent,

(c) every two good choice sets of S are recursively equivalent.

If the md-class S is infinite, (a) and (b) need no longer be true. For let S contain infinitely many sets of cardinality $\geqq 2$, e.g., $S=((0, 1), (2, 3), (4, 5), \cdots)$. Then S has c choice sets. Every good choice set of S has the form $p(\sigma)$ for some partial recursive function $p(x)$, hence S has at most $\aleph_0$ good choice sets and (a) is false. Every nonzero RET contains exactly $\aleph_0$ sets; the c choice sets of S can therefore not all be recursively equivalent and (b) is false. On the

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