IRREDUCIBLE SEMIGROUPS

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Introduction. Let S be a compact connected semigroup with identity, which we denote by 1. If S is not a group, then there is a proper unique minimal ideal $M \subset S$ which is itself compact and connected, and hence also there is a minimal compact connected subsemigroup containing the identity and meeting M. Such a semigroup we call *irreducible*. (The property of being irreducible is independent of the semigroup in which such a semigroup is contained.)[2] Because every compact connected semigroup with identity which is not a group contains an irreducible semigroup joining its identity to its minimal ideal, the study of these is of utmost importance and a knowledge of their structure appears to be more or less an optimal expectation in the direction of a general structure theorem for compact semigroups (although in particular cases, of course, more can be expected). It is the purpose of this note to announce the structure of several large classes of irreducible semigroups (which in particular includes the abelian ones, a case in which quite a bit is already known from the work of Hunter [5]). The proof is quite lengthy and will appear in our forthcoming book.

The structure of the maximal subgroup of the identity, being a compact group, and of the minimal ideal, being a *Rees product* of groups [10], is known. Attempts to move out into the remaining portion have been successful in particular cases [2], [3], [7], the degree of success being to a large extent proportional to the success in finding the structure of an irreducible subsemigroup joining the identity and the minimal ideal. In every previous case in which this structure has been found, the irreducible semigroup has been an I-semigroup (i.e., a semigroup with identity on an arc such that the minimal ideal reduces to one end point [7]) or the closure of a one-parameter subsemigroup [1]. However, examples of much more complicated irreducible semigroups have been given by Hunter [4], and some information about these semigroups was obtained by him and Rothman [6] and by Rothman [8], [9], mainly in the case of *normal* semi-

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[2] An equivalent definition is: a semigroup is irreducible if it is compact, connected, has an identity, and contains no proper compact connected subsemigroup containing its identity and meeting its minimal ideal. In the terminology of Hunter and Rothman, this is a semigroup *algebraically* irreducible between the identity and some idempotent in the minimal ideal.