

## REFERENCES

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**DECOMPOSITION OF RIEMANNIAN MANIFOLDS<sup>1</sup>**

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A decomposition problem in geometry is the following: given a  $C^\infty$  manifold  $M$  on which is defined an affine connection without torsion, under what conditions in terms of the holonomy group will  $M$  be affinely diffeomorphic to a direct product of two other affinely connected manifolds? Here, we give a complete solution to the problem in the case the connection is one induced by a (definite or indefinite) riemannian metric—Main Theorem.

For a more detailed discussion of this general problem, see [1] and [2], especially §5.1–5.3 of [1], §I of [2], as well as Theorem 1 below. Clarifications of various concepts introduced in this paper are also given therein.

We need some definitions. Inner products on vector spaces and riemannian metrics on manifolds can be either definite or indefinite in this paper. A subspace  $V'$  of an inner-product space  $V$  is said to be nondegenerate (resp., degenerate, isotropic) iff the restriction of the inner product to  $V'$  is nondegenerate (resp., degenerate, zero). The action of a connected Lie group  $G$  acting on  $V$  will be said to be nondegenerately reducible iff  $G$  leaves invariant a proper nondegenerate subspace of  $V$ . The maximal subspace of  $V$  on which  $G$  acts as the identity is called the maximal trivial space of  $G$  in  $V$ . From now on, we fix a point  $m \in M$ . Then the holonomy group of  $M$  will always be understood to be acting on  $M_m$  (tangent space to  $M$  at  $m$ ), so that in this case all references to  $M_m$  will be omitted. Finally, a pair  $(\phi, M^1 \times M^2)$  is called an affine decomposition (resp. an isometric decomposition) of the affinely connected manifold  $M$  (resp. rieman-

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