THE GEOMETRY OF IMMERSIONS. II

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This announcement is the sequel to the announcement *Geometry of immersions*. I, which appeared in the September 1963 issue of this journal. In the present announcement we develop the characteristic class theory of Thom (see [6], [7]) to include some situations of differential geometric interest. We then apply these techniques of differential topology to some problems of extrinsic differential geometry. Most notable among them is §6 which deals with the problem of counting umbilic points on an immersed hypersurface.

In this announcement we draw freely on the definitions contained in our first announcement. Full details will appear in a separate publication.

1. *G*-structures on vector bundles. Their singularities and characteristic classes.

DEFINITION 1.1. Let $\phi_1: G\to GL(p, R)$ and $\phi_2: H\to GL(q, R)$ be faithful smooth representations of the Lie groups G and H in $GL(p, R)$ and $GL(q, R)$ respectively. This induces in a natural way an action of $G\times H$ on $\mathrm{Hom}(R^p, R^q)$. Let K be a regular manifold collection of submanifolds of $\mathrm{Hom}(R^p, R^q)$. Then K is called a *model $G\times H$ singularity with respect to ϕ_1 and ϕ_2*, if K is invariant under the action of $G\times H$ on $\mathrm{Hom}(R^p, R^q)$.

DEFINITION 1.2. Let $\xi=(\pi: E\to X)$ be a smooth vector bundle of fiber dimension q. Let $\pi_E: P_E\to X$ be the principal $GL(q, R)$ bundle associated to E. Let G be a Lie group. Let $\phi: G\to GL(q, R)$ be a faithful representation of G. A *(G, ϕ)-structure on E* is given by a principal G-bundle $\pi_G: P_G\to X$, and a smooth mapping $\tilde{\phi}: P_G\to P_E$ such that

(a) $\pi_E\cdot\tilde{\phi}=\pi_G$,

(b) $\tilde{\phi}$ is smooth and 1-1,

(c) for each $u\in P_G$ and $g\in G$, $\tilde{\phi}(u)\phi(g)=\tilde{\phi}(ug)$.

REMARK. A (G, ϕ)-structure on a vector bundle $\xi=(\pi: E\to X)$ of fiber dimension q, gives rise to a reduction of the structural group $GL(q, R)$ to $\phi(G)$. Hence the representation ϕ allows us to consider the vector bundle ξ as a vector bundle associated with the principal G-bundle P_G, because the G action on the fibers will be linear.

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