ON THE SYMMETRY OF CONVEX BODIES

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Communicated by Victor Klee, February 9, 1964

We say that a convex body in n-dimensional Euclidean space E_n is "k-symmetric" if it coincides with its reflection through some k-plane. Let K be an n-dimensional convex body and K' a k-symmetric convex body of maximum volume contained in K. Define

$$c(K;\,k) = \frac{V(K')}{V(K)},$$

where $V(K)$ is the volume of K. Let

$$c(n,\,k) = \inf\{c(K;\,k)\colon K \subset E_n\}.$$

THEOREM 1.

$$c(n,\,k) \geqq \frac{\max\{k!,\,(n-k)!\}}{2^{n-k}n!}, \qquad 0 \leqq k < n.$$

This generalizes the result, $c(n,\,0) > 2^{-n}$, proved in [**3**].

One can also consider K as a nonhomogeneous solid with density $f(p)$ at each $p \in K$, and ask for a symmetric subset of maximum mass. Restricting ourselves to the case of 0-symmetry (i.e., central symmetry), we define for each integrable density f on K

$$\mu(K;f) = \frac{M(K')}{M(K)},$$

where K' is a centrally symmetric convex body of maximum mass contained in K, and $M(K)$ is the mass of K. Let $\mu(K)$ be the infimum of $\mu(K;f)$, for f ranging over all integrable densities, and define

$$\mu(n) = \inf\{\mu(K)\colon K \subset E_n\}.$$

THEOREM 2. *$\mu(n) \geqq 2^{-n}$, $n \geqq 3$, and $\mu(2) = 1/3$.*

The first inequality follows from an obvious generalization of the computation of "mean symmetry" used in [**3**], while the second equality depends on the fact (see Theorem 4) that any plane convex body is the union of 3 centrally symmetric convex bodies.

Let $g(n)$ be the least number r such that any n-dimensional convex body K can be covered by r translates of $-K$ (equivalently, $g(n)$ is the least number r such that any n-dimensional convex body is the