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A SPARSE REGULAR SEQUENCE OF EXPONENTIALS CLOSED ON LARGE SETS

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Introduction. For a given sequence $\{\lambda_k\}$ of complex numbers, the problem of determining those intervals I on which the exponentials $\{e^{i\lambda_k x}\}$ are complete in various function spaces has been extensively studied [3]. Since the problem is invariant under a translation of I, only the lengths of I are involved, and attention has focused on the relation between these lengths and the density of the sequence $\{\lambda_k\}$. With the function space taken to be $L^p(I)$ for $1 \leqq p < \infty$, or $C(I)$, the continuous functions on I, the general character of the results has been that there exist sparse real sequences (lim r^{-1} (the number of $|\lambda_k| < r) = 0$, for example) for which I can be arbitrarily long [2], but all such sequences are nonuniformly distributed; when a sequence is sufficiently regular, in the sense that λ_k is close enough to k, the length of I cannot exceed 2π [4, p. 210]. Most recently, in a complete solution which accounts for all these phenomena, Beurling and Malliavin have proved that the supremum of the lengths of I is proportional to an appropriately defined density of $\{\lambda_k\}$ [1].

The purpose of this note is to show that the situation is quite different when the single interval I is replaced by a union of intervals. Specifically, we will construct a real symmetric (or positive) sequence $\{\lambda_k\}$ arbitrarily close to the integers, for which the corresponding exponentials are complete in $C(S)$, where S is any finite union of the intervals $|x - 2n\pi| < \pi - \delta$, with integer n and $\delta > 0$, and so has arbitrarily large measure. Thus, for sets S more general than intervals,