ON A STATIONARY APPROACH TO SCATTERING PROBLEM[1]

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Communicated by P. Lax, April 1, 1964

1. Let H_p, $p=1, 2$, be self-adjoint operators in a Hilbert space $\mathfrak{H}$ satisfying the condition

$$(1)\qquad (H_1 - z)^{-1} - (H_0 - z)^{-1} \in \boldsymbol{T}(\mathfrak{H}), \qquad z \in \rho(H_0) \cap \rho(H_1).$$

Here, $\boldsymbol{T}(\mathfrak{H})$ denotes the trace class of completely continuous operators in $\mathfrak{H}$ and $\rho(H_p)$ the resolvent set of H_p. The perturbation theory of absolutely continuous (abbr. a.c.) parts of H_p as well as the theory of wave and scattering operators has recently been studied independently by de Branges [2], Birman and Kreĭn [1], and Kato [3]. In [1] and [3] the problem was considered from the viewpoint of the scattering theory. In particular, the wave operators $W_\pm$ were proved to exist and hence to be partially isometric operators which give the unitary equivalence of a.c. parts of H_0 and H_1. In [2], on the contrary, similar partially isometric operators $\hat{W}_\pm$ were constructed somewhat explicitly and *without* referring to the limit of wave operator type. The purpose of the present note is to study the latter approach from a viewpoint of the scattering theory and to see that the so-called time-independent or stationary approach to the theory of wave and scattering operators can be made possible under the condition (1). In a simpler case, a similar study was made in [4]. Our construction of the operator similar to $\hat{W}_\pm$, i.e. the operator given by the right side of (9), is similar to but slightly different from that given in [2]. In particular, the use of the auxiliary operator I in [2] is avoided. Furthermore, the construction of the operators π_0 and π_1 in 3 might be a little more explicit than that of the corresponding operators given in [2].

2. Let $\mathfrak{C}$ be a separable Hilbert space and let $\boldsymbol{T}_p \equiv \boldsymbol{T}_p(\mathfrak{C}) \subset \boldsymbol{T}(\mathfrak{C})$ be the set of all non-negative operators in $\boldsymbol{T}(\mathfrak{C})$. The trace norm will generally be denoted by $\tau(\ )$. Let μ be a $\boldsymbol{T}_p$-valued measure defined for bounded Borel sets of the reals R^1. Then the set function ρ, first defined at each bounded Borel set e as $\rho(e) = \tau(\mu(e))$ and then ex-

[1] The work was partly supported by the National Science Foundation through Purdue University (NSF G-18920). The author wishes to express his thanks to Professor Louis de Branges for valuable discussions and kind hospitality extended to the author while he was visiting Purdue University.