ON THE LOCAL BEHAVIOR OF THE RATIONAL TSCHEBYSCHEFF OPERATOR

BY HELMUT WERNER

Communicated by A. S. Householder, February 10, 1964

Let l and r be non-negative integers. Denote by $\mathcal{R}_{l,r}$ the set of all rational functions where the degrees of the numerator and denominator do not exceed l and r respectively. If $R=p/q\in\mathcal{R}_{l,r}$ and p and q are relatively prime polynomials of degree ∂p and ∂q, then $d_{l,r}[R]:=\min\,[l-\partial p,\ r-\partial q]$ is called the defect of R in $\mathcal{R}_{l,r}$: the function R is called degenerate, if the defect is positive. (For these notations compare Werner (1962) [**3**].)

For a fixed interval $[a, b]$ let $T_{l,r}[f]$ be the Tschebyscheff Approximation of $f\in C[a, b]$ in the class $\mathcal{R}_{l,r}$ with respect to the norm $\|f\|:=\max_{[a,b]}|w(x)\cdot f(x)|$, with $w(x)$ a positive continuous weight function in $[a, b]$. We write $\eta_{l,r}[f]:=\|f-T_{l,r}[f]\|$. Those f for which $T_{l,r}[f]$ is not degenerate are called normal by Cheney and Loeb (1963) [**1**]. Already Maehly and Witzgall (1960) [**2**] proved that $T_{l,r}[f]$ furnishes a continuous map of $C[a, b]$ into itself at f with respect to the introduced norm, if f is normal. For the actual verification of normality one may use the following *normality criterion*:

Let $g(x)$ be normal for $T_{l,r}$. Then $f(x)$ is normal if

$$\|f-g\| < (\eta_{l-1,r-1}[g] - \eta_{l,r}[g])/2.$$

Except for the case $r=1$, l arbitrary (compare Werner (1963) [**5**]) no specific properties of f are known to insure normality of f for arbitrary l, r.[1] Maehly and Witzgall (1960) [**2**] also gave an example that showed that $T_{l,r}[f]$ need not be continuous at f, if f is not normal. Recently Cheney and Loeb (1963) [**1**] made an extensive study of generalized rational approximation and proved that $T_{l,r}[f]$ is not continuous, if f is not normal and if no alternant of the error function $\eta(x):=w(x)(f(x)-T_{l,r}[f](x))$ has $r+l+2$ points. This later restriction may be lifted and one obtains the following classification.

THEOREM 1. *The operator $T_{l,r}[f]$ is continuous at f if and only if f is normal or belongs to the class $\mathcal{R}_{l,r}$.*

In order to prove this, one now only has to cope with the case that the error function has an alternant of $l+r+2$ points. By a proper

[1] *Added in proof.* Recently a criterion has been published by H. L. Loeb, Notices Amer. Math. Soc. 11 (1964), 335.