EIGENFUNCTION EXPANSIONS AND SCATTERING THEORY FOR PERTURBED ELLIPTIC PARTIAL DIFFERENTIAL OPERATORS[1]

BY PETER C. GREINER

Communicated by F. Browder, March 19, 1964

1. A number of papers discussing the spectral decomposition and eigenfunction expansion for partial differential operators appeared in the last few years. Browder [1], [2], [3], [4], Gårding [5] and Mautner [12] proved the existence of an abstract eigenfunction expansion for elliptic partial differential operators. In 1953 A. Ya. Povzner [13] considered the detailed spectral decomposition of $-\Delta+q(x)$. This was completed by T. Ikebe [6] who used the theory of wave operators as developed by Kato [8] and Kuroda [10], [11].

In this note we investigate an eigenfunction expansion for the operator $P(D)+q(x)$ where $P(D)$ is a linear homogeneous elliptic partial differential operator with constant coefficients. Detailed proofs of the results will appear elsewhere.

2. The Euclidean n-space will be denoted by R_n or M_n with elements $x=(x_1, \cdots, x_n)$ or $k=(k_1, \cdots, k_n)$ respectively. $\int f(x)dx$ denotes integration with respect to Lebesgue measure. We set

$$-D_j = \frac{\partial}{i\partial x_j} \qquad \text{for } 1 \leqq j \leqq n.$$

Let $P(x)$ be a homogeneous elliptic polynomial, i.e. $P(x)\geqq c|x|^{2p}$ where $2p$ is the order of $P(x)$. Then $P(D)=P(D_1, \cdots, D_n)$ is a linear homogeneous elliptic partial differential operator. All through this note we assume that $4p>n$. It is well known that $P(D)$ can be extended to a selfadjoint operator $\tilde{P}(D)$ in $L_2(R_n)$. Let $q(x)\in C_{2[n/2]}$ with $q(x)=O(|x|^{-n-h})$ for some $h>0$. Then by Theorem 1 of [11], $\tilde{P}(D)+q(x)$ is a selfadjoint operator in $L_2(R_n)$. Let $\{E_t\}$ and $\{P_t\}$, $-\infty<t<+\infty$, be the resolutions of the identity for $\tilde{P}(D)$ and $\tilde{P}(D)+q(x)$ respectively. Define

[1] This research was partially supported by the National Science Foundation under Grant No. NSF-GP-2283. The results reported in this note are contained in a dissertation submitted in partial fulfillment of the requirements for the Ph.D. at Yale University. We are happy to have this opportunity to express our gratitude to Professor Felix E. Browder for his generous advice and many helpful conversations during the preparation of this paper.