RESEARCH PROBLEMS

6. E. T. Parker: *Finite combinatorial set theory.*

Let a, b, c be positive integers, $a>b>c$. Given a set S of a elements and a class K of n distinct subsets of b elements each from S, there must exist three distinct sets in K having at least c elements in common, provided n is sufficiently large. Develop upper and lower bounds on $n(a, b, c)$. (The *three* sets can of course be generalized to $t>1$.) (Received January 30, 1964.)

7. G. B. Dantzig: *Eight unsolved problems from mathematical programming.*

a. Let C_n be an n-dimensional bounded polyhedral convex set defined by $2n$ distinct faces, n of which determine the extreme point p_1 and the remaining n of which determine the extreme point p_2. Does there always exist a chain of edges joining p_1 to p_2 such that the number of edges in the chain is n?

b. Let E be the extreme points of a unit n-cube having as faces the coordinate hyperplanes through the origin and the hyperplanes parallel to them passing through the point $(1, 1, \cdots, 1)$. Let P be a given hyperplane which separates E into two parts E_1 and E_2. Characterize the $n-1$ dimensional faces of the convex set having E_1 as its complete set of extreme points.

c. A matrix such that the determinant of every square submatrix has value -1, 0, or $+1$ is called *unimodular*. The distinct columns of such a matrix are said to form a *complete set* if the annexation of a column not in the set destroys the unimodular property. Two such sets *belong to the same class* if one can be obtained from the other by a permutation of the rows of its matrix. Characterize the various classes. How can they be generated? Given a matrix, find necessary and sufficient conditions that it satisfy the unimodular property.

d. Given an $n\times n$ permutation matrix $[x_{ij}]$, i.e., a zero-one matrix each row and column of which have exactly one unit element. Let the set of its n^2 elements constitute a point in n^2-dimensional coordinate space. It is known that these and only these points are extreme points of the convex set defined by

$$\sum_{i=1}^{n} x_{ij} = 1, \qquad j = 1, 2, \cdots, n,$$

$$\sum_{j=1}^{n} x_{ij} = 1, \qquad i = 1, 2, \cdots, n, \qquad x_{ij} \geqq 0.$$