BOOK REVIEWS

Cohomology operations. Lectures by N. E. Steenrod. Written and revised by D. B. A. Epstein. Annals of Mathematics Studies, No. 50. Princeton University Press, Princeton, N. J., 1962. 139 pp. $3.00.

Cohomology operations have had, since their discovery in the 1940's, many important applications in algebraic and differential topology. In spite of this, there has been essentially no source outside of the original papers for the study of these operations. The book under review here represents a major step towards filling this gap. It treats the Steenrod reduced powers, giving a new improved method of construction (which does not appear in print elsewhere) together with many of their interesting applications.

The book begins with a set of axioms for the Steenrod algebra $\mathcal{A}(2)$, postponing the existence and uniqueness to the final chapters. The vector space basis of Adem ("Relations on Steenrod Powers of Cohomology Classes," *Algebraic geometry and topology*, Princeton, 1957) and Cartan (*Sur l'iteration des operations de Steenrod*, Comment. Math. Helv. **29** (1955), 40–58) is obtained and it is shown that the indecomposable elements of $\mathcal{A}(2)$ are of the form $S_q^{2^i}$. This second fact is shown to put severe restrictions on the kinds of truncated polynomial rings that can occur as the mod 2 cohomology ring of a space and also to show that, if $f\colon S^{2n-1}\to S^n$ has odd Hopf invariant, then n is a power of 2. It is also shown that, for n even, maps $S^{2n-1}\to S^n$ exist with any even Hopf invariant.

In Chapter 2, the structure of $\mathcal{A}(2)$ as a Hopf algebra is studied and the results of Milnor (*The Steenrod algebra and its dual*, Ann. of Math. (2) **67** (1958), 150–171) on the dual Hopf algebra $\mathcal{A}(2)^*$ are obtained. In Chapter 3, the nonembedding theorems of Thom and Hopf are proved, Thom's theorem dealing with the embeddability of a compact space in a sphere and Hopf's theorem with the embeddability of an $(n-1)$-manifold in an n-sphere.

Chapter 4 is devoted to the determination of the cohomology rings of the classical groups and the Stiefel manifolds. This is done by exhibiting an explicit cellular structure for these spaces. The action of $\mathcal{A}(2)$ on the mod 2 cohomology of the real Stiefel manifolds is derived and the results applied to obtain an upper bound for the number of linearly independent tangent vector fields on a sphere.

The next chapter develops some of the technical machinery needed in the construction of the reduced powers. The notion of equivariant cohomology is introduced and the cohomology of a group is defined.