

WHAT THE BEES KNOW AND WHAT THEY DO NOT KNOW

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In the first part of this paper we construct a more economical honeycomb than that of the hive bees for any parameters involved in the problem. The second part gives a survey of some further solved and unsolved isoperimetric problems concerning cell-aggregates.

I. Honeycombs. The honeycomb of the bees is a loose tissue of wax forming a plane layer. The first things on it which catch one's eye are the regular hexagonal patterns on both sides. The hexagons are the openings of prismatic vessels, called bee-cells. Kepler described the shape of the bee-cells more fully. It turned out that the bottom of a cell consists of three equal rhombi as shown in Figure 1b. Thus the two kinds of cells having their openings in opposite directions are separated by a zigzagged surface and not by a plane, as one would expect at first.

Why do the bees build such a strange conformation?

According to a widely spread hypothesis, going back to Pappus, the bees aim at economy: If, by some reason, the volume of a cell and the width of the whole layer are given, they try to use the minimum amount of wax per cell. Although among the various effects which interact in producing the honeycomb the utilitarian human motive attributed to the bees seems to play the least part, the above hypothesis was the source of highly interesting investigations. Thus we accept this hypothesis and try to point out what the bees do well and what they do not do well from the point of view of making the surface-area of their cells small.

To give the problem a precise formulation we define a *honeycomb* as a set of congruent convex polyhedra, called *cells*, filling the space between two parallel planes without overlapping and without interstices in such a way that

(1) Each cell has a face, called *base* (or *opening*) on one of the two planes but does not have faces on both planes.

(2) In the congruence of two cells their bases correspond to each other.

The distance between the parallel planes is the *width* of the honeycomb.

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