# HARMONIC ANALYSIS AND THE THEORY OF COCHAINS

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1. Let $E^2$ represent the plane endowed with the usual Cartesian coordinate system, and let $R$ be an open set contained in $E^2$. We say that $X$ is a 1-cochain defined in $R$ (see [7, p. 5]) if (a) $X(\sigma)$ is a real number for every 1-simplex $\sigma$ (i.e., oriented line segment) contained in $R$, (b) $X(-\sigma)=-X(\sigma)$ for every 1-simplex $\sigma$ contained in $R$, (c) $X(\sigma)=X(\sigma_1)+\cdots+X(\sigma_n)$ for $\sigma=\sigma_1+\cdots+\sigma_n$ with $\sigma, \sigma_1, \cdots, \sigma_n$ collinear, similarly oriented, and contained in $R$. $X$ is then extended by linearity to all chains in $R$; so in particular if $\tau$ is a 2-simplex (i.e., oriented triangle), $X(\partial\tau)$ is defined.

We shall call the 1-cochain $X$ a local $L^1$ 1-cochain in $R$ if the following two conditions are met:

(1) there exist two non-negative functions $g_1(x)$ and $g_2(y)$, each locally in $L^1$ on $R$ such that

($\alpha$) if $\sigma$ is a 1-simplex in $R$ parallel to and oriented like the $x$-axis, $|X(\sigma)| \leqq \int_\sigma g_1(x)dx$,

($\beta$) if $\sigma$ is a 1-simplex in $R$ parallel to and oriented like the $y$-axis, $|X(\sigma)| \leqq \int_\sigma g_2(y)dy$;

(2) there exists a non-negative function $H(x, y)$ locally in $L^1$ on $R$ such that if $\tau$ is a 2-simplex oriented like $E^2$ with two edges parallel to the $x$ and $y$-axes and $\tau$ is in $R$, then

$$|X(\partial\tau)| \leqq \int_\tau H(x, y)dxdy.$$

Let $Q$ be a measurable set contained in $R$ with the property that $|R-Q|_2=0$ (where $|\ \ |_j$ represents $j$-dimensional Lebesgue measure). Using the notation of [7, p. 262], we say that the 1-simplex $\sigma$ in $R$ is $Q$-good if $|\sigma-(\sigma\cap Q)|_1=0$. We say that a 2-simplex $\tau$ contained in $R$ is $Q$-excellent if each of the 1-simplices in $\partial\tau$ are $Q$-good.

We shall call the differential form $\omega(x, y)=a(x, y)dx+b(x, y)dy$ a local $L^1$ differential 1-form in $R$ if the following three conditions are met:

(3) $a(x, y)$ and $b(x, y)$ are measurable functions in $R$;

(4) there exists a measurable set $Q\subset R$ with $|R-Q|_2=0$ and two

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