IRREDUCIBLE GRAPHS

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Communicated January 22, 1964

A graph G is *n-irreducible* if it *cannot* be imbedded in an orientable 2-manifold M of genus $(n-1)$, but, for *any* arc a in G, the graph G_a, obtained by removing a from G, *can* be imbedded in M.

A classical result of Kuratowski states that there are essentially only two 1-irreducible graphs. These are K_5, the complete 5-graph (five vertices and all possible connecting arcs), and $K_{3,3}$, a hexagon with its "long" diagonals.

Practically nothing is known about n-irreducible graphs for $n>1$. For example, it is not even known whether the collection of 2-irreducible graphs is finite. (See [2, p. 63].)

Under the circumstances it is interesting to observe that n-irreducible graphs for $n>1$ can be constructed easily as a consequence of one of the principal results in [1].

In order to conserve space the reader is referred to [3] for basic definitions and background information.

The first result is a useful and simple theorem characterizing n-irreducibility.

THEOREM. *A graph G is n-irreducible if and only if $\gamma(G)=n$ and $\gamma(G_a)=(n-1)$ for any arc a in G.*

PROOF. If G is n-irreducible, then, by definition,

$$\gamma(G) \geqq n, \tag{1}$$

$$\gamma(G_a) \leqq (n-1). \tag{2}$$

Suppose M is an orientable 2-manifold of genus $\gamma(G_a)$. Then G_a can be imbedded in M. By adding a suitably located "handle" to M one can accommodate the arc a missing from G; in short, imbed G. The new 2-manifold has genus one larger than M. This fact, together with (1) and (2), implies

$$n \leqq \gamma(G) \leqq \gamma(G_a) + 1 \leqq n.$$

Consequently equality signs hold throughout, and

$$\gamma(G) = n$$

$$\gamma(G_a) = (n-1).$$