INEQUALITIES FOR GENERAL MATRIX FUNCTIONS

BY MARVIN MARCUS[1] AND HENRYK MINC[2]

Communicated by A. S. Householder, November 22, 1963

I. **Introduction.** In [4] Schur proved the following beautiful result. If H is a subgroup of the symmetric group of degree m, S_m, and $\chi(\sigma)$ is a character of degree 1 of H, then

$$\det A \leqq \sum_{\sigma \in H} \chi(\sigma) \prod_{t=1}^{m} a_{t\sigma(t)} \tag{1}$$

for any m-square positive semi-definite hermitian matrix A. Observe that if H is the identity group, the inequality (1) is the Hadamard determinant theorem $\det A \leqq \prod_{i=1}^m a_{ii}$. In [3] it was conjectured that $\text{per } A \geqq \prod_{i=1}^m a_{ii}$ and in [2] this inequality was proved. Here $\text{per } A = \sum_{\sigma \in S_m} \prod_{t=1}^m a_{t\sigma(t)}$ is the permanent of A.

The purpose of the present paper is to announce some inequalities for the general matrix function

$$d_\chi(A) = \sum_{\sigma \in H} \chi(\sigma) \prod_{t=1}^{m} a_{t\sigma(t)}. \tag{2}$$

We shall see subsequently that Schur's inequality (1) is an immediate corollary to our Theorem 4.

II. **Main results.**

THEOREM 1. *If N is m-square normal with characteristic roots $\eta_1, \cdots, \eta_m$, then*

$$\left| d_\chi(N) \right| \leqq \frac{1}{m} \sum_{i=1}^{m} \left| \eta_i \right|^m. \tag{3}$$

In case $\chi \equiv 1$, we have the following generalization of the van der Waerden conjecture in the non-negative hermitian case [3; 5].

THEOREM 2. *Let A be an m-square positive semi-definite hermitian. Let the ith row sum of A be denoted by r_i, $i = 1, \cdots, m$, and suppose $\sum_{i=1}^m r_i = r > 0$. Then*

[1] The work of this author was supported by the National Science Foundation under grant G. P. 1085.

[2] The work of this author was supported by the Air Force Office of Scientific Re search.